



Knowledge, Development and Quantities of Public and Private Schools in Modern Society

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

This paper demonstrates that rationality of an economic agent is closely related to the expertise and related knowledge, and personal attainment is generally positively related with education. We believe that social development levels are based on the corresponding knowledge and education levels. We analyzed the different functions of private and public education, the price formation of public education, student's academic achievement, and the quantity of public and private schools. We demonstrated the importance of education market and university ranking in the development of education, and analyzed the proportion of people engaged in education in a modern economy.

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1. INTRODUCTION

Natural science makes people know more and deeper about the universe, and technology enables people make new products, explore more and more deep into the outer space, and social science enable people establish cost-efficient economic and political institutions. In a modern society, generally, student learns knowledge at public or private schools.

Usually an economic agent is supposed to be rational in modern society, which means a person will maximize his utilities or profit, but this may not be true if he does not have adequate expertise and related knowledge. For example, a child cannot maximize his long-run utility before he grows up and a young man cannot operate a complex machine without professional training.

In the competition for survival, such as in a war, the people with more expertise and physical power will win, and that is why we establish military schools in modern society. Education aims to impart expertise and related knowledge to students, enables them master necessary expertise and related knowledge and skills, making them more rational in a certain discipline.

Rationality of an economic agent is generally positively related with received expertise and related knowledge. Homogeneity will be high if gaps of received knowledge and education among people are small, and heterogeneity will be high if gaps of received knowledge and education among people are great. A market may be more efficient if all economic agents are at the same level of received expertise and education other things being equal. The relationship of rationality with received expertise and related knowledge is indicated in Fig. 1, and the relationship of market efficiency with knowledge gap is indicated in Fig. 2.

Then, a question arises, where does knowledge come from, and who will take research of the urgent problems that people are eagerly confronting with. In early societies, it is old people who have accumulated valuable experience and impart them to the youth, such as skills of making instruments, hunting, planting and so on. Gradually some experts established schools teaching students.

Early formal schools were documented in ancient Greece, China and India, though nobody knows

exactly when the first school was created. Xiong [1] indicated that there are government-established schools in the West Zhou Dynasty in China, schools are affiliated to and controlled by governments, names, ranks of schools affiliated to the central and local governments are different (1046-771 B.C.). Yang [2] indicated that Confucius is one of the first people who established private schools in Spring and Autumn Period in China (about 520 B.C.). Table 1 indicates the top 10 oldest schools in the world.

Some scholars oppose conventional styles of schooling, indicated that the conventional schooling would engulf the life time of a student, educational expenditure was a burden to students, and suggested that de-institutionalizing education may be a starting point for a de-institutionalized society [4]. Hern [5] indicated that there is no right way to educate a child, the conventional education deadens children's natural love for learning, undercuts their self-esteem, and limits independent thought. He suggested replacing compulsory schooling with a wide variety of home, neighborhood and community-based education.

The conventional education system has shortcomings, and reforms have been made to meet the demands of students. A school is a place where experts conduct research and teach students. A school is also an authority and a third party that assures us that a student who has received a diploma is well educated, and school is the symbol of institutional progress. School itself is not a problem, and the problem is what kind of school we need. We need to trade off the benefit and cost of schools, we can reduce schools to a necessary quantity, but adequate schools will do benefit to our society.

The outline of the study is organized as follows: Section 1 is introduction. The relationship of career attainments and education is analyzed in section 2. The relationship of social development with knowledge and education is analyzed in section 3. Different functions of private and public education are analyzed in section 4. In section 5, student's academic achievement, price formation of public education and the quantity of public and private schools are analyzed, and we emphasized the importance of education market and university ranking in the development of education. We present a model analyzing the proportion of people engaged in education in section 6. Section 7 is discussion and Conclusion.

Table 1. The oldest schools in the world: Top 10

Name	Location	Time	Type	Country
Chengdu Shishi High School	Chengdu	143–141 B.C.	Public school	China
The King's School,	Canterbury,	597 A.D	Private school	England
The King's School,	Rochester	604 A.D.	Private school	England
St Peter's School	York	627 A.D	Private school	England
Thetford Grammar School	Thetford	631 A.D.	Private school	England
Hereford Cathedral School	Hereford	676 A.D.	Private school	England
Royal Grammar School Worcester	Worcester	685 A.D	Private school	England
Beverley Grammar School,	Beverley	700 A.D.	state school	England
Gymnasium Paulinum	Münster	797 A. D	a cathedral school	Germany
Gymnasium Theodorianum	Paderborn	799 A.D.	Public school	Germany

Resource: Wikipedia, List of the oldest schools in the world [3] and related website

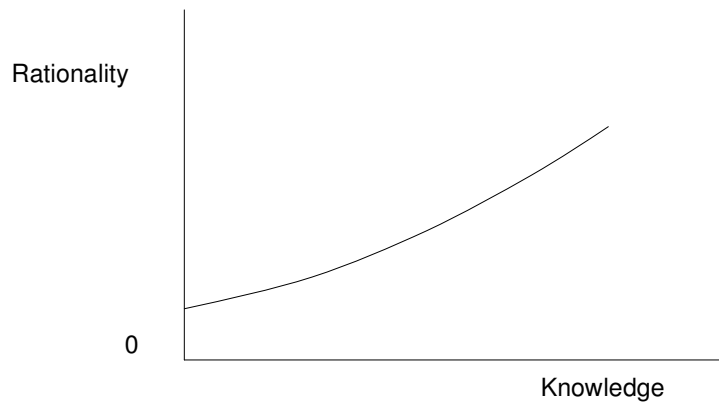


Fig. 1. The relationship of rationality and knowledge

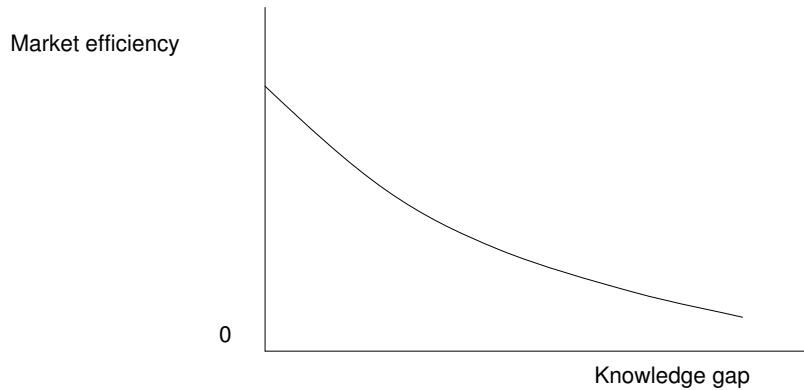


Fig. 2. The relationship of market efficiency and knowledge gap

2. CAREER ATTAINMENTS AND EDUCATION

2.1 Entrepreneurship and Education

Advanced business education can increase the propensity of people toward entrepreneurship, formal education can increase entrepreneurial efficiency and successful firm growth, the

number of years of formal education increase the probability of becoming self-employed [6]. On-the-job training may be more important than the quality of the training [7], expertise in business is fundamental to the survival and success of small business [8].

Personality traits are partly developed by innate nurturing, socialization and education [9].

Education system have a crucial role in developing the entrepreneurial traits of students, for example, curricula are very important in the developing of independence, creativity and risk taking [10].

Some socialists [11] indicated that the influence of education on being an entrepreneur is not linear. Black and Smith [12] found that the effect of quality of college on returns to education diminished as other factors were included, such as ability and demographics, and they find that attending an elite university increase the wages of men by 11 percent and women by 7.5 percent.

Loury and Gardman [13] found that selectivity did not play a very important role on the returns to education. Goedhuys and Sleuwaegen [14] indicated that primary education does not have a significant impact on being an entrepreneur, but higher education has a significant impact on being an entrepreneur.

A higher level of educational attainment does not necessarily increase risk taking propensity of students, but a higher level of educational attainment can provides an individual with more confidence to become entrepreneurially active, a student with a higher level of education has a higher intention to become an entrepreneur [15].

Wu and Wu [16] reported that respondents with a diploma and undergraduate degree show higher interest in starting a business than those with a post graduate degree. According to the researchers, individuals with higher levels of education may have many options, which decrease the intention to entrepreneurship of students [15].

2.2 Career Attainments and Education

Education had a positive effect on professionalism, overall performance and organizational commitment [17]. Education has a positive effect on career development [18] and elites tend to have more higher education in developed countries [19]. A master's degree has a larger positive effect on promotion probability and on the rate of salary increases [20]. Managers with a bachelor's degree earned substantially less than managers with a master's degree [21]. Degrees above masters have also been found to add significantly to future earnings, career attainment will be positively related to number of years of schooling [22].

Many corporate leaders come from a few top ranked schools, graduating from an Ivy League school can increase a manager' salary [21]. The more selective a school, the more likely a student will receive a high salary [23]. Attending an elite private institution will generate increased economic returns [24]. Baruch and Leeming [25] indicated that return from educational attainment is significant, such as salary, promotion, satisfaction level with a job.

In July 2002, the US Census Bureau indicated that a high school graduate can get an average salary of USD 18.900, a college graduate can get an average salary of USD 45.400, and a graduate with a professional degree can get an average salary of USD 99.300 [18]. The rates of return to education vary overtime, but education is becoming more and more important in career attainment in recent years [26].

In OECD countries, people received tertiary education have the highest employment rate. The unemployment rate of people received an upper secondary education is 13%, and unemployment rate of people received the tertiary education is 5% [27].

2.3 Curriculum, Social Culture and Career Attainments

Curriculum is important factor that influences attainment. Loury and Gardman [13] found that college majors had a significantly large effect on earnings. Larkins [28] found that a bachelors' degree of science and technology yield greater returns than humanities and social science. College majors associated with greater career attainment are engineering and business [29]. Spilerman and Lunde [20] found that the probability with an engineering major of being promoted was high and it is positively associated with promotion rate.

Career attainment is also affected by other variables, such as campus culture, family culture, economic condition and geographic location [24]. School education may not be the direct cause of salary differentials, but employers are interested in hiring intelligent individuals [30]. Effect of quality of college on returns to education was influenced by many individual factors, such as ability and demographics [12].

3. KNOWLEDGE, EDUCATION AND SOCIAL DEVELOPMENT

Generally, education is a form of learning, in which the knowledge, skills, and habits of a group of people are transferred from one generation to the next through teaching, training, or research [31]. World scientific culture has provided a strong support for education expansion, and worldly wide competition promotes economic development of nation countries [32,33]. Goodlad and McMannon [34] indicated that the youths must be enculturated to comply with social and political expectations, such as civility and Civitas.

Weber [35] indicated that the origins of industrial revolution can be traced to the Protestant reformation and the rise of Calvinism. Different religions and culture are key factors that determine the economic performance and equilibrium outcomes for a given set of institutions, such as beliefs, world views, values and preferences of a society [36].

Primary education is taken as an investment for the social and economic development in 1950s by some economists, such as Schultz [37] and it is taken as a basic universal human right in the 1970s [38]. Education is viewed as a most efficacious instrument for modernization by developing countries [39]. The United Nations Educational, Scientific and Cultural Organization (UNESCO) was established in 1945 to promote intercultural understanding, long lasting peace and development, UNESCO takes education as a fundamental human right and a prerequisite for human development, and every child has the right to quality education. Economic and political agreement may not adequately guarantee the long lasting order and peace, and order and peace must be established on the basis of humanity's moral and intellectual solidarity, which is one of the aims of the UNESCO.

3.1 Productivity and Education

Human capital is a necessity to the social development, such as the training of general education, firm-specific training and relevant experience [21]. Schultz [40] introduced human capital into the production function to resolve the residual puzzle to explain economic growth. Education can enhance productivity and make an individual earn more money, so education is important to development [41].

Education can promote economic growth, its

influence may not be prompt, but it will show its effect at later times. The study of Peaslee [42] suggests that there may be a lag of 8 years for the tertiary education to play a role, and a lag of 12 years for the secondary education. But the lag may be short in present days because of the information technology such as internet.

A certain quantitative education is requisite for economic development, but education alone does not assure economic development, other economic factors, behavioral factors, political factors exert varying influences to economic development [42]. It is increasingly recognized that knowledge and innovation are key to sustaining economic growth, high level of skills and knowledge can create and fuel knowledge based economies.

An average of 83% of today's young people in OECD countries will complete upper secondary education over their lifetimes. 79% of young people will complete upper secondary education over their lifetimes in G20 countries. On average, 60% of young adults (67% of women and 53% of men) in OECD countries are expected to enter tertiary-type A program over their life,¹ 3% are expected to enter advanced research programs, 19% of today's young adults (20% of women and 18% of men) will enter tertiary-type B programs over their lifetimes [27].²

3.2 Human Capital and Education

It is acknowledged conventionally that education is a central ingredient of economic development, and can produce valuable economic returns [32]. Smith [43] indicated that education can promote productive efficiency, as well as individual and social development, Walsh [44] took education as an investment because the physical input including land, machines and labors cannot account for all the output.

Educational (or manpower) planning is a central ingredient of economic development. Inkeles [45] indicated that education is directly related with modernization in six developing countries. Education is more effective than other factors on

¹ Programs designed to provide qualifications for entry to advanced research, such as medicine, dentistry or architecture. Duration is at least 3 years full-time. Tertiary-type A programs include second-degree programs, such as the American master's degree.

² Programs are typically shorter than those of tertiary-type A and focus on practical, technical or occupational skills for direct entry into the labor market, and they have a minimum duration of two years full-time equivalent at the tertiary level.

modernization, such as work experience or urbanization [46,47]. The study of Peaslee [42] demonstrates that there is a close relationship between economy and high primary education enrollments. The relationship between illiteracy and poverty, hunger and disease throughout the world is accepted increasingly [48].

Education is a social investment, and can produce valuable economic returns [32]. Private returns on investment in tertiary education are comparatively substantial, it pays off the expenditure of an individual, and the public also benefit in the form of tax revenues and social contributions. The net public return of tertiary education of a man is over USD 100 000 across OECD countries, it is about three times the amount of public investment in education. The public return of a women is around USD 60 000, it is about twice the amount of public investment [27].

3.3 Economic, Political Institutions and Education

Education create and legitimate modern citizen and elite roles in the political system, and emphasize the responsibility of citizenry, political development and education are reciprocally related [49]. Modern societies are schooled, based on the allocation of authority of education

and allocate political rights and capacities to people with appropriate credentials [4].

National educational systems have expanded rapidly after the Second World War [50], Contemporary educational systems are politically constructed institutions, and they are structured by political authorities to incorporate human resources. The federal government of USA passed the first federal statutes and established the first federal office of education during and after the Civil War, it is seen as a way to consolidate its position in the South [51,52]. Communist states and other totalitarian states control higher education to form and legitimate the political center and authority [53,54].

We know that some basic rules have to be implemented to insure the cooperation of the society, such as ethics, norms and laws. And a third party is important to implement the rules and punishment in case that some person violates the rules. A third party may be a court, a department of the government that implements the rules and punishment [55]. Laws and third parties are important components of economic and political institutions, and knowledge and education promote the establishment of the institutions. Fig. 3 indicates the development of culture and institutions along with that of knowledge and education.

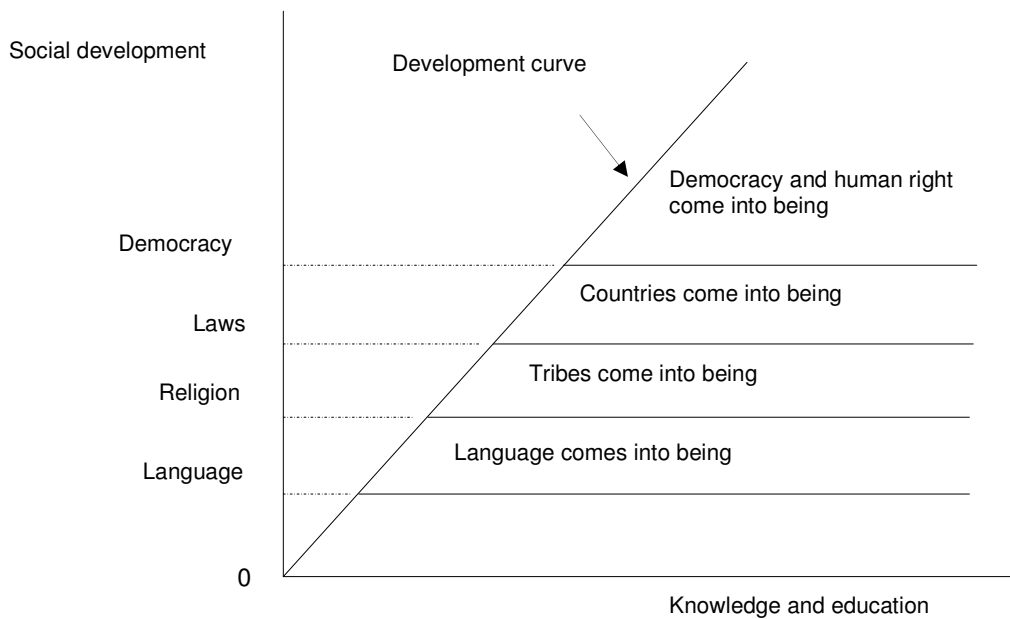


Fig. 3. The Relationship of Social development with knowledge and education

4. DIFFERENT FUNCTION OF PRIVATE AND PUBLIC EDUCATION

Public goods are non-excludable and non-subtractable [56], or non-excludable and non-rival [57]. Obviously, knowledge has the two characteristics of public goods, and we take it as public good. Knowledge makes education an impure public good although education can be private managed [58]. Benjamin R. Barber [34] indicated that schools are public because they serve public purposes and multicultural education is essential to maintain democracy. Of course, linguistic pluralism may be impedimental to universal literacy [48].

McCulloch indicated that a better law of inheritance and a better system of education are two powerful instruments to reduce inequality of income [59]. Public education reduces income inequality more quickly than private education when social income inequality is large, and private education is more prone to higher per capita income [60].

Private schooling leads to higher growth when the gap of human capital endowments across families is small, and public education has higher growth when the gap of human capital is big. Public schooling leads to income convergence, while private schooling results in ever increasing inequality [61]. So public education plays a role of redistribution of social income, and the amount of public schools should increase when income gap increases.

4.1 Private Education can Enlarge Income Gap

We know that investment in education of a child is a question of wealth distribution between two generations. Suppose that there are two persons in a family, a father and a son, they have a two-period problem: in the first period, the father has to make a decision whether he will let the boy go to school. The boy will get a normal job in the second period if he is well educated in the first period; he will get a low income job in the second period if he is not well educated or not educated at all. A normal job will be paid a normal salary of b regardless of the periods. The boy will get a salary of b if he gets a normal job in the second period, and he will get a salary of $\frac{b}{n}$ if he gets a low income job, $n=1, 2, \dots$ Suppose the probability that the boy is well educated is p , the cost of education is c , the rate of bank interest is r , so we have an equation:

$$p(b-c(1+r))+(1-p)\left(\frac{b}{n}-c(1+r)\right)=\frac{b}{n} \quad (1)$$

$$\Rightarrow c=\frac{bp\left(1-\frac{1}{n}\right)}{1+r}$$

We can see that c is positively related with the normal income b , the probability p , and the income gap $\left(1-\frac{1}{n}\right)$, and negatively related with the rate of bank interest.

Suppose at time t , there is a proportion α_0 of families that have an income lower than c , they cannot afford the tuition fee. Then the proportion of families that can afford tuition fees is $1-\alpha_0$. We have supposed that the probability a boy get well educated is p , then at time $t+1$, the proportion of children who are not well educated is $(1-\alpha_0)(1-p)$. The not educated plus not well educated is $\alpha_0+(1-\alpha_0)(1-p)$, these children will have low income jobs. Suppose a small proportion β of those children who have low income jobs at time t can get normal jobs at time $t+1$, then the proportion of children who have low income jobs at time $t+1$ will be $\alpha_0+(1-\alpha_0)(1-p)-\alpha_0\beta$, and it is $(1-\beta)\alpha_0+(1-\alpha_0)(1-p)$. Then we have the recursion formula:

$$a_{n+1}=(1-\beta)a_n+(1-a_n)(1-p) \quad (2)$$

$$\Rightarrow a_{n+1}=(p-\beta)a_n+1-p$$

$$a_{n+1}=\begin{cases} \text{increase, if } p > \beta \\ 1-p, \text{ if } p = \beta \\ \text{decrease if } p < \beta \end{cases}$$

The series will be convergent if $p \leq \beta$, the series will be divergent if $p > \beta$, which means the proportion of children who have a low income jobs will increase. Generally in an undeveloped economy, p is larger than β , the children who have low income jobs will increase.

Suppose a_n has been the biggest, and $a_{n+1}=a_n$, and from $a_{n+1}=(p-\beta)a_n+1-p$, we can get $a_n=\frac{1-p}{1-p+\beta}$, so the maximum value is $\frac{1-p}{1-p+\beta}$, the proportion of children with low income jobs is included in the region: $(\alpha_0, \frac{1-p}{1-p+\beta})$.

$$\frac{1-p}{1-p+\beta}=\begin{cases} 0 \text{ if } p = 1 \\ 1 \text{ if } p = 0 \text{ (and } \beta = 0) \\ (0, 1) \text{ if } 0 < p < 1 \end{cases}$$

So we have a conclusion: (1) at the equilibrium point, the proportion of children with low income

jobs is determined by p and β , it is not determined by the first value α_0 ; and (2) the proportion of children with low income jobs will approach 0 if p approaches 1, and the proportion will approach 1 if p approach 0 (under the condition $\beta = 0$).

4.2 The Role of Public Education

Compulsory education plays a role of redistributing social resources, enhance welfare and make distribution more even in a society [62]. Minimum schooling laws will decrease the inequality of income distribution [63] and the majority of individuals will be better off under a certain level of compulsory education in the long run. Theodore Sizer indicated that public education will give every child intellectual and civic tools, and enable children to have decent lives in modern culture and economy [34].

Public education is a way to counteract or weaken the mechanism of private education that enlarges income gap. Private education can enlarge income gap, and we can lessen the income gap by providing public education to children from poor families. Free access to basic public education may provide the only chance to overcome poverty when parental human capital is low. By contrast, at advanced stages of development, free public education crowds out private education investment if parental human capital is high, and stimulates fertility and may impede growth [64].

In England, compulsory education was taken in 1870 by the Elementary Education Act [65], Frederic the Great of Prussia began compulsory elementary education in 1763, and Maria Theresa of Austria began compulsory elementary education in 1773 [66]. Having children in school for a specified period was adopted in the eighteenth century, and were followed by other countries, the majority of countries adopted compulsory basic education after the Second World War [67]. In the 20th century, compulsory basic education is unanimously accepted as an essential basic public service, and the compulsory education law is widely observed and enforced in all countries [66]. In U.S.A, the state universities and land-grant colleges are very good enterprises, they are the base of the democratic system and the well being of the economy and society [68].

5. QUANTITY OF PUBLIC AND PRIVATE SCHOOLS

5.1 School Level Effect, Family Background, Public Policy and Student's Academic Achievement

School policy, discipline and the different sets of incentives directly influence the academic achievement of students, family background and public policy is also very important to the academic achievement of students.

The study of Coleman, Hoffer and Kilgore [69] demonstrates that Catholic and other private schools have higher achievement than public schools, and academic, disciplinary policies, academic levels of students and their families can account for this. On average, private school students have higher scores in mathematics achievement than public schools in the eighth grade in the twelve national educational regions of Thailand plus Bangkok and unit costs in private schools are also much lower than public schools. Classroom practices and peer group characteristics can account for substantial part of the difference [70].

Students from public-school than private-school of Yale seem more likely to go on to do doctoral work, and place more emphasis on academic achievement, and private-school students seems to place more emphasis on the accumulation of social capital, this was also true for the men of Harvard University and the women of Radcliffe College [71].

To test the mathematical achievement at public and private schools, Sarah Theule Lubienski and Christopher Lubienski [72] conducted a study with students at grade 4 and grade 8. As a whole, the average mathematics achievement of private schools was higher than public schools. Then they employ the method of quartile according to socioeconomic status of students, within each quartile of socioeconomic status, the average mathematics achievement of the public schools is higher than private schools at both grade 4 and 8, socioeconomic status causes academic difference: less than 40% of public schools were of high socioeconomic status, over 80% of private schools were of high socioeconomic status.

Lubienski, Crane and Lubienski [73] furthered the study on academic achievements of students and indicated: after controlling for demographic

difference, on average, initial kindergarten achievement of public school is nearly identical with Catholic schools, but lower than the private schools; after controlling for demographic difference and kindergarten scores, student's mathematics achievements at fifth grade at public schools is higher than Catholic schools, basically identical with the private schools. All variables in the model explained 62% of the achievement differences between schools, school type alone accounted for less than 5%, family background accounted for the most.

Jensen [74] indicated that higher level student academic performance in Catholic schools was mainly caused by the greater discipline and more rigorous requirements. Peer group characteristics can account for a substantial part of the student's academic difference [70]. University self-regulation is the best way to assure academic standard, such as the Teacher Education Accreditation Council (TEAC), Teaching-Learning-Quality Process Reviews (TLQPR) [75].

Majority voting will determine the education quality of schools, technically, the quality of a school is determined by the marginal return and the marginal cost to raise the education standard, the quality will be improved if the marginal return is greater than the marginal cost to raise education standard of the school. Such as USA and Italy, the USA produces a system with high quality of private schools and low quality of public schools, while Italy produce a system with high quality of public schools and low quality of private schools [76].

5.2 Price of Public Education

Knowledge is public good, and it has the characteristic of scale economy and strong external effect, this enables education have the characteristic of strong external effect and scale economy. Scale economy means marginal cost will decrease along with the increase of output, and the per capita education price will be lower if one person (or corporation) manages the supply of public education than two persons, but one person means monopoly, and the market power of monopoly will lift the price. To take the advantage of scale economy, and avoid the disadvantage of monopoly, the managing of public education should be publicly bid.

Technically, within the design capacity, the marginal cost of education will decline, and the per capita education price will decline. Beyond the design capacity, marginal cost will increase, and the effect of scale economy will disappear.

Generally the quantity of pupils who need the public education is approximately given, and different education managers have different marginal cost and bidding price. Fig. 4 indicates the price formation of public education, y_0 is the total demand of education, it is approximately given. p_1 is the winning bid price, p_0 is the marginal cost. y_c is the design capacity, beyond point C, the marginal cost will increase.

The number of managers who compete to manage public education, and that of pupils who need the public education, are two important factors that influence the price of public education.

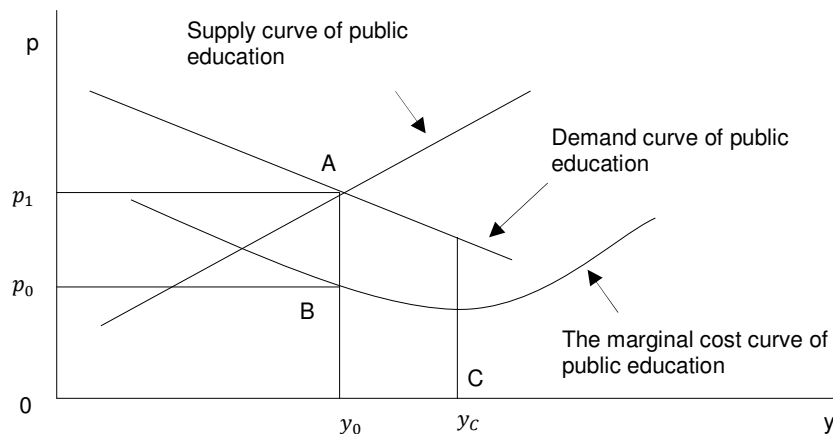


Fig. 4. Price formation of public education

5.3 Quantity of Public and Private Schools

Egalitarianism in education may be a utopia, and it will lower the schooling standard, elitism may provide students with the best professionals in a variety fields, and it will get a good return [77].

Limited public spending, cultural heterogeneity, especially religious heterogeneity is major reasons that promote the development of private schools [78]. On-the-job training, expertise in business also promotes the development of private schools.

Non-public schools may have higher efficiency and effectiveness, the total schooling cost per student is lower than that in public schools, but non-public schools may only provide students with the easy and cheaper aspects of education, do not provide more special needs [77]. Maybe schools are more suitable for private management along with the increase of age [77].

Suppose the combination of public schools and private schools is optimal, the output of the combination will cultivate the most qualified students. Suppose public schools are not profit making, and private schools are profit making. Tuition fees of public schools are low (or no tuition fees), tuition fees of private schools are high, teachers and education resources can flow freely at the education market.

Suppose the combination of public schools and private schools is like a Cobb-Douglas function, public school and private school are two factors, the total investment of public schools and private schools is a constant, and we want the optimum quantities of public schools and private schools. If the total investment is m , then we have the following optimum question:

Max (Y)

$$\text{s. t. } Y = Ax_1^\alpha x_2^\beta \quad (3)$$

$$p_1 x_1 + p_2 x_2 = m \quad (4)$$

x_1 is the quantity of public schools, and x_2 is the quantity of private schools, $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$, p_1 is the tuition fee of public schools, p_2 is the tuition fee of private schools, the total investment of education (or endowment) is m . We have the answers of x_1 and x_2 :

$$x_1 = \frac{\alpha m}{(\alpha + \beta)p_1}, \quad x_2 = \frac{\beta m}{(\alpha + \beta)p_2}$$

Suppose $\alpha + \beta = 1$, we have: $x_1 = \frac{\alpha m}{p_1}$, $x_2 = \frac{\beta m}{p_2}$

We can see that the quantities of public and private schools are positively related with their income in the economy and negatively related with the tuition fee.

Suppose that the public schools aim to meet the basic demand of education, they cannot enlarge or shut down in the short run. In the contrary, private schools will meet a variety of demands of the education market, a new school may be established, and an old school may be shut down. Suppose the growth rate of private schools is a , $1 \leq a \leq 1$, the added education income is $p_2 a x_2$, and the budget of total education is extended to $m + p_2 a x_2$, and the question is:

Max (Y)

$$\text{s. t. } Y = Ax_1^\alpha x_2^\beta$$

$$p_1 x_1 + p_2 x_2 = m + p_2 a x_2$$

$$\text{We have: } x_1 = \frac{\alpha m}{(\alpha + \beta)p_1}, \quad x_2 = \frac{\beta m}{(\alpha + \beta)(1 - a)p_2}$$

Suppose $\alpha + \beta = 1$, we have: $x_1 = \frac{\alpha m}{p_1}$, $x_2 = \frac{\beta m}{(1 - a)p_2}$

In a poor economy, people do not have much resource to invest in private education, and public schools may be the main way to get science and technology. With the development of economy, people have more resource to invest in private education, and different kinks of private schools can be built to satisfy different demands of people. The development curves of private and public education is indicated in Fig. 5.

Public education has the role of redistribution of income, so it should be positively related with income gap. Fig. 6 indicates the relationship of public education with income gap.

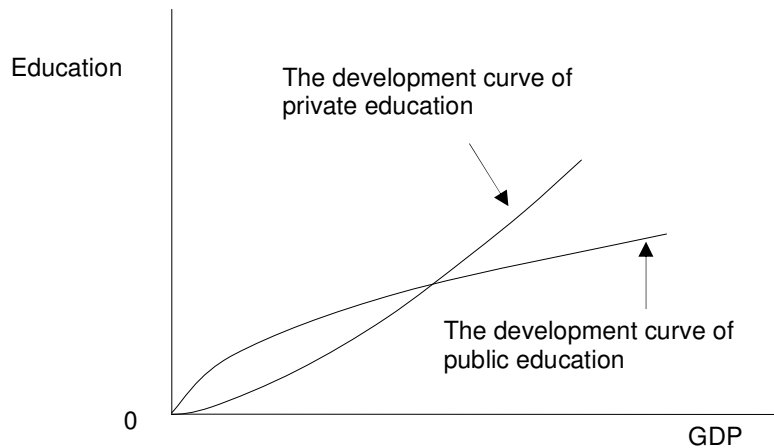


Fig. 5. Development curves of public and private education

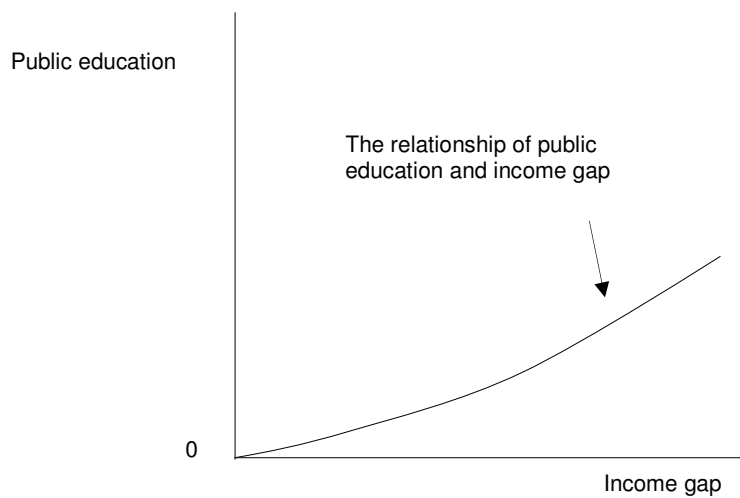


Fig. 6. The relationship of public education and income gap

In most countries, public schools provide education from primary education to tertiary education. On average across OECD countries in 2011, about 89% of primary students, 86% of lower secondary students and 81% of upper secondary students are enrolled in public schools. Slightly less than 3% of primary students are enrolled in fully private schools, slightly more than 3% of lower secondary and more than 5% of upper secondary are enrolled in private schools. 15% of students of tertiary type A and advanced research programs are enrolled in private schools, and the proportion will be 29% if government dependent private schools are included. 20% of students of tertiary type B programs are enrolled in private schools and the

proportion will be 41% if government dependent private schools are included [27].

The United Kingdom is the only country that 100% of students of tertiary-type A and advanced research programs are enrolled in government dependent private schools (or institutions), so do the students of tertiary-type B programs [27]. But Williams [79] indicated that the entire university system in the United Kingdom is composed of autonomous, property-owning schools, and they are independent by Royal Charter or by Parliamentary Statute, so the universities of the United Kingdom is more similar to the non-profit private universities of the U.S.A., other than state-controlled universities.

5.4 Education Market and University Ranking

Chubb and Moe [80] indicated that public schools are bureaucratic, and they are governed by institutions of direct democratic control, institutional inertia may prevent effective organizational adaptation to changing environmental conditions [81]. Peterson [82] indicated that public schools made a noncompetitive environment and monopoly position, and it seemed that public schools were in a flourish.

Some research public universities in the U.S.A. and state-controlled universities in many other countries are actively disconnecting themselves from the control of government both by academic activities and management, and the deregulation of research universities from state agencies to public corporations is in the public interest [83]. Distinction between private and public universities is becoming blurred, and public policies play an important role to assure all publicly subsidized universities to provide human capital to society with efficiency and equity, regardless of what type of school they are [75].

Market competition for being more selective and prestigious may divert school resources away from improving student's academic standard, but for-profit universities are more likely to take resources to improve program and services to meet the needs of student than public and not-for-profit schools [75,84].

The QS (Quacquarelli Symonds) World University Rankings ranks universities annually, it provide rankings according to academic

reputation, employer reputation, faculty student, international faculty, international students and citations per faculty. The latest ranking in 2013 of the world's top universities has been released by the QS. The first is Massachusetts Institute of Technology (MIT), continues to rank first as last year, the second is Harvard University, the third is The University of Cambridge. Table 2 presents the world top 10 universities, the top 10 are dominated by universities of the U.S.A. and the United Kingdom.

Market competition is an important force that furthers the development of education, an open, free, and fair market is a necessary platform that develops education. Obviously, university ranking is a good way to encourage schools to engage in education and research, just like the Nobel Prize being a way to encourage experts to achieve great academic success.

6. PROPORTION OF PEOPLE ENGAGED IN EDUCATION

6.1 Proportion of People Engaged in Education

Towns will be full of discontent students, urban slums and delinquents increase when students from primary school are more than jobs [53], Dore [86] took the over competition for education as diploma disease, people worried about over education, and schools may be in excessive supply. Universal education on the style of present schools will engulf the lifetime of students, it may not be feasible any longer, and educational webs may be alternate ways [4].

Table 2. The top 10 universities in the world 2013

Rank	University	Type	Country
1	Massachusetts Institute of Technology	private research university	U.S.A
2	Harvard University	private research university	U.S.A
3	University of Cambridge	public collegiate research university (the colleges are private)	United Kingdom
4	UCL (University College London)	public research university	United Kingdom
5	Imperial College London	public research university	United Kingdom
6	University of Oxford	collegiate research university	United Kingdom
7	Stanford University	private research university	U.S.A
8	Yale University	private research university	U.S.A
9	University of Chicago	private research university	U.S.A
10	California Institute of Technology	private research university	U.S.A
10	Princeton University	private research university	U.S.A

Sources: QS TOP UNIVERSITIES, QS World University Rankings 2013 [85], and related website

Chevalier [87] indicated that investment in education will cease at the point that the current value of future income equals to the cost of education. In a market economy, how many schools are needed is determined by the demand of education market, and education prices will solve the problem. Here we want to know the proportion of people who engaged in education, and the growing trend.

Suppose there are two sectors in an economy, one is labor, the other is education. Labor provides labor force, education provides knowledge. The optimum combination of them will produce the maximum output. Suppose the combination is like a Cobb-Douglas function, then we have the following function.

$$Y_t = AE_t^\alpha r_t^\beta L_t^\beta \quad (5)$$

We take E as education, which means knowledge comes from education, and L as labor. Y_t is the output at time t, E_t is the education at time t, r_t is the proportion of people engaged in labor at time t, L_t is the total available labor force of an economy at time t. $A > 0$, and A is a coefficient. Suppose E_t changes according to the following function.

$$\dot{E}_t + \delta E_t = BL_t(1-r_t) \quad (6)$$

\dot{E}_t is the change of education, it is an abbreviation of $\frac{dE_t}{dt}$, δ is a discount factor, it means that old knowledge may depreciate if new knowledge comes into being. ρ is a discount factor, it is the rate that we discount future income to present value. $B > 0$, and B is a coefficient, just like A. The question is indicated in the following functions.

$$\text{Max } \sum_{t=0}^T e^{-\rho t} \ln(Y_t)$$

$$\text{s. t. } Y_t = AE_t^\alpha r_t^\beta L_t^\beta$$

$$\dot{E}_t + \delta E_t = BL_t(1-r_t)$$

And we have the answers of r_t and $1-r_t$:

$$r_t = \frac{(\delta + \rho)\beta}{\alpha\delta + (\delta + \rho)\beta}, \quad 1-r_t = \frac{\alpha\delta}{\alpha\delta + (\delta + \rho)\beta}$$

The proportion of people who engage in education is $1-r_t = \frac{\alpha\delta}{\alpha\delta + (\delta + \rho)\beta}$, the procedure of solving the function is in appendix I.

We can see that the proportion of people engaged in education is getting bigger and bigger, α is getting bigger and bigger, if knowledge is becoming more and more important.

6.2 Proportion of People Engaged in Research and Education

Suppose research produce knowledge and education imparts knowledge. There are three sectors in an economy, and they are research, education and labor. The optimum combination of them will produce the maximum output. Suppose the combination is like a Cobb-Douglas function, then we have the following function.

$$Y_t = AR_t^\alpha E_t^\beta r_{0t}^\gamma L_t^\gamma \quad (7)$$

And education and research changes according to the following functions respectively:

$$\dot{E}_t + \delta_1 E_t = B_1 L_t r_{1t} \quad (8)$$

$$\dot{R}_t + \delta_2 R_t = B_2 L_t (1-r_{0t}-r_{1t}) \quad (9)$$

The dynamic optimization question is:

$$\text{Max } \sum_{t=0}^T e^{-\rho t} \ln(Y_t)$$

$$\text{s. t. } Y_t = AR_t^\alpha E_t^\beta r_{0t}^\gamma L_t^\gamma$$

$$\dot{E}_t + \delta_1 E_t = B_1 L_t r_{1t}$$

$$\dot{R}_t + \delta_2 R_t = B_2 L_t (1-r_{0t}-r_{1t})$$

R_t is the research at time t, E_t is the education at time t, L_t is the labor at time t. \dot{E}_t is the change of education at time t, it is the abbreviation of $\frac{dE_t}{dt}$, \dot{R}_t is the change of research at time t, it is the abbreviation of $\frac{dR_t}{dt}$. r_{0t} is the proportion of people engaged in labor, r_{1t} is the proportion of people engaged in education, $1-r_{0t}-r_{1t}$ is the proportion of people engaged in research. δ_1 , δ_2 and ρ are discount factors, B_1 and B_2 are coefficient, just like A. We have the answers of r_{0t} , r_{1t} and $1-r_{0t}-r_{1t}$:

$$r_{0t} = \frac{(\rho + \delta_1)(\rho + \delta_2)\gamma}{(\rho + \delta_1)\alpha\delta_2 + (\rho + \delta_2)\beta\delta_1 + (\rho + \delta_2)(\rho + \delta_1)\gamma}$$

$$r_{1t} = \frac{(\rho + \delta_2)\beta\delta_1}{(\rho + \delta_1)\alpha\delta_2 + (\rho + \delta_2)\beta\delta_1 + (\rho + \delta_2)(\rho + \delta_1)\gamma}$$

$$1-r_{0t}-r_{1t} = \frac{(\rho + \delta_1)\alpha\delta_2}{(\rho + \delta_1)\alpha\delta_2 + (\rho + \delta_2)\beta\delta_1 + (\rho + \delta_2)(\rho + \delta_1)\gamma}$$

The procedure of solving the equation is indicated in appendix II.

Research is getting more and more important if new knowledge becomes more and more important, and there will be more and more people engaged in research and education.

7. DISCUSSION AND CONCLUSION

Resource per capita is becoming increasingly scarce on the earth along with the increase of population, such as the global warming. New science and technology is more and more important to increase productivity and to solve pollution problems. Education aims to produce and impart knowledge [88], so education is becoming more and more important. A child cannot receive education if his family is poor, and he cannot find a satisfying job when he grows up, and this is not fair, so public education should at least meet the demand of elementary education of children from poor families.

Religion and faith are important content of education, political institutions of a country is established on certain religion and faith, so education is mutually related with political institutions, and government wants to control education by funding schools and education policy. So the development of social knowledge and social institutions seems more difficult than science and technology, this may prevent the social development, such as some totalitarian states.

Schools are competing for students, teachers and dollars, and market force is taken as a panacea for problems of the public schools [89] and market pressures have produced organizational changes in public community colleges [90]. Private schools have significant impact on the performance of public schools in states with large private school sectors, and public school students have improved education performance [81].

De-schooling may be extreme, but it is good to simplify schooling, and it will reduce the burden of students. What kind of knowledge and how many schools we need are very difficult questions, and these questions should be solved by education market, of course, education needs the support of public policy.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX I

$$\text{Max } \sum_{t=0}^T e^{-\rho t} \ln(Y_t)$$

$$\text{s. t. } Y_t = A E_t^\alpha r_t^\beta L_t^\beta$$

$$\dot{E}_t + \delta E_t = B L_t (1 - r_t)$$

To solve the equation, we need to construct a Hamilton equation:

$$H = e^{-\rho t} (\ln(A E_t^\alpha r_t^\beta L_t^\beta) + \mu_t (B L_t (1 - r_t) - \delta E_t)) \quad (1)$$

The following is the process of solving the equation

$$\begin{aligned} \frac{\partial H}{\partial r_t} &= e^{-\rho t} \left(\frac{\beta}{r_t} - \mu_t B L_t \right) = 0 \\ &\Rightarrow \frac{\beta}{r_t} = \mu_t B L_t \end{aligned} \quad (2)$$

$$\text{From } \frac{\beta}{r_t} = \mu_t B L_t, \text{ we have: } \frac{\dot{r}_t}{r_t} = -\frac{\dot{\mu}_t}{\mu_t} \quad (3)$$

$$\frac{\partial H}{\partial E_t} = e^{-\rho t} \left(\frac{\alpha}{E_t} - \mu_t \delta \right)$$

According to Euler Equation, we have

$$\begin{aligned} \frac{d(e^{-\rho t} \mu_t)}{dt} &= -\frac{\partial H}{\partial E_t} \Rightarrow -\rho e^{-\rho t} \mu_t + e^{-\rho t} \frac{d\mu_t}{dt} = -e^{-\rho t} \left(\frac{\alpha}{E_t} - \mu_t \delta \right) \\ &\Rightarrow -\rho \mu_t + \frac{d\mu_t}{dt} = -\frac{\alpha}{E_t} + \mu_t \delta, \text{ or } -\rho \mu_t + \dot{\mu}_t = -\frac{\alpha}{E_t} + \mu_t \delta \\ &\Rightarrow \dot{\mu}_t = -\frac{\alpha}{E_t} + (\delta + \rho) \mu_t \end{aligned} \quad (4)$$

Replace $\dot{\mu}_t$ with \dot{r}_t (equation (3)), and replace μ_t with r_t (equation (2)), we have:

$$-\frac{\dot{r}_t}{r_t} \mu_t = -\frac{\alpha}{E_t} + (\delta + \rho) \mu_t \Rightarrow -\frac{\dot{r}_t}{r_t} \frac{\beta}{B L_t r_t} = -\frac{\alpha}{E_t} + (\delta + \rho) \frac{\beta}{B L_t r_t} \Rightarrow \dot{r}_t = \left(\frac{\alpha}{E_t} - (\delta + \rho) \frac{\beta}{B L_t r_t} \right) \frac{\beta r_t}{B L_t r_t}$$

We have two equations:

$$\dot{r}_t = \left(\frac{\alpha}{E_t} - (\delta + \rho) \frac{\beta}{B L_t r_t} \right) \frac{\beta r_t}{B L_t r_t}$$

$$\dot{E}_t + \delta E_t = B L_t (1 - r_t)$$

At an equilibrium point, we suppose $\dot{r}_t = 0$, $\dot{E}_t = 0$, so we have:

$$0 = \left(\frac{\alpha}{E_t} - (\delta + \rho) \frac{\beta}{B L_t r_t} \right) \frac{\beta r_t}{B L_t r_t} \quad (5)$$

$$\delta E_t = B L_t (1 - r_t) \quad (6)$$

And we have:

$$r_t = \frac{(\delta + \rho) \beta}{\alpha \delta + (\delta + \rho) \beta}$$

APPENDIX II

$$\text{Max } \sum_{t=0}^T e^{-\rho t} \ln(Y_t)$$

$$\text{s. t. } Y_t = AR_t^\alpha E_t^\beta r_{0t}^\gamma L_t^\gamma$$

$$\dot{E}_t + \delta_1 E_t = B_1 L_t r_{1t}$$

$$\dot{R}_t + \delta_2 R_t = B_2 L_t (1 - r_{0t} - r_{1t})$$

We need to construct a Hamilton equation to solve the question:

$$H = e^{-\rho t} (\ln (AR_t^\alpha E_t^\beta r_{0t}^\gamma L_t^\gamma)) + \mu_{1t} (B_1 L_t r_{1t} - \delta_1 E_t) + \mu_{2t} (B_2 L_t (1 - r_{0t} - r_{1t}) - \delta_2 R_t) \quad (1)$$

$$\frac{\partial H}{\partial r_{0t}} = e^{-\rho t} \left(\frac{\gamma}{r_{0t}} - \mu_{2t} B_2 L_t \right) = 0$$

$$\Rightarrow \frac{\gamma}{r_{0t}} = \mu_{2t} B_2 L_t \quad (2)$$

Suppose $\mu_{1t}' = \frac{d\mu_{1t}}{dt}$, $\mu_{2t}' = \frac{d\mu_{2t}}{dt}$, and we have:

$$\Rightarrow \frac{\gamma}{r_{0t}} = \mu_{2t} B_2 L_t \Rightarrow -\frac{r_{0t}'}{r_{0t}^2} = \frac{\mu_{2t}'}{\mu_{2t}} \quad (3)$$

$$\frac{\partial H}{\partial r_{1t}} = e^{-\rho t} (\mu_{1t} B_1 L_t - \mu_{2t} B_2 L_t) = 0 \Rightarrow \mu_{1t} B_1 L_t = \mu_{2t} B_2 L_t$$

$$\Rightarrow \frac{\mu_{1t}'}{\mu_{1t}} = \frac{\mu_{2t}'}{\mu_{2t}} \quad (4)$$

From equation (3) and (4), we have:

$$\frac{\mu_{1t}'}{\mu_{1t}} = \frac{\mu_{2t}'}{\mu_{2t}} = -\frac{r_{0t}'}{r_{0t}} \quad (5)$$

$$\frac{d(e^{-\rho t} \mu_{1t})}{dt} = -\rho e^{-\rho t} \mu_{1t} + e^{-\rho t} \frac{d\mu_{1t}}{dt} \quad (6)$$

$$\frac{d(e^{-\rho t} \mu_{2t})}{dt} = -\rho e^{-\rho t} \mu_{2t} + e^{-\rho t} \frac{d\mu_{2t}}{dt} \quad (7)$$

$$\frac{\partial H}{\partial E_t} = e^{-\rho t} \left(\frac{\beta}{E_t} - \mu_{1t} \delta_1 \right) \quad (8)$$

$$\frac{\partial H}{\partial R_t} = e^{-\rho t} \left(\frac{\alpha}{R_t} - \mu_{2t} \delta_2 \right) \quad (9)$$

According to Euler equation, we have:

$$\frac{d(e^{-\rho t} \mu_{1t})}{dt} = -\frac{\partial H}{\partial E_t} \Rightarrow -\rho e^{-\rho t} \mu_{1t} + e^{-\rho t} \frac{d\mu_{1t}}{dt} = -e^{-\rho t} \left(\frac{\beta}{E_t} - \mu_{1t} \delta_1 \right)$$

$$\Rightarrow -\rho \mu_{1t} + \frac{d\mu_{1t}}{dt} = -\left(\frac{\beta}{E_t} - \mu_{1t} \delta_1 \right) \Rightarrow -\rho \mu_{1t} + \frac{d\mu_{1t}}{dt} = \mu_{1t} \delta_1 - \frac{\beta}{E_t}$$

$$-\rho \mu_{1t} + \mu_{1t}' = \mu_{1t} \delta_1 - \frac{\beta}{E_t}$$

$$\dot{\mu}_{1t} = (\rho + \delta_1)\mu_{1t} - \frac{\beta}{E_t} \quad (10)$$

According to Euler equation, we have:

$$\begin{aligned} \frac{d(e^{-\rho t}\mu_{2t})}{dt} &= -\frac{\partial H}{\partial R_t} \Rightarrow \rho e^{-\rho t}\mu_{2t} + e^{-\rho t}\frac{d\mu_{2t}}{dt} = -e^{-\rho t}\left(\frac{\alpha}{R_t} - \mu_{2t}\delta_2\right) \\ \Rightarrow -\rho\mu_{2t} + \frac{d\mu_{2t}}{dt} &= -\left(\frac{\alpha}{R_t} - \mu_{2t}\delta_2\right) \Rightarrow -\rho\mu_{2t} + \frac{d\mu_{2t}}{dt} = -\left(\frac{\alpha}{R_t} - \mu_{2t}\delta_2\right) \\ -\rho\mu_{2t} + \dot{\mu}_{2t} &= -\left(\frac{\alpha}{R_t} - \mu_{2t}\delta_2\right) \Rightarrow -\rho\mu_{2t} + \dot{\mu}_{2t} = \mu_{2t}\delta_2 - \frac{\alpha}{R_t} \\ \dot{\mu}_{2t} &= (\delta_2 + \rho)\mu_{2t} - \frac{\alpha}{R_t} \end{aligned} \quad (11)$$

From equation (2), (3), (4) and (5), we have:

$$\begin{aligned} \dot{\mu}_{2t} &= -\frac{\dot{r}_{0t}}{r_{0t}}\mu_{2t}, \dot{\mu}_{1t} = -\frac{\dot{r}_{0t}}{r_{0t}}\mu_{1t} \\ \mu_{2t} &= \frac{Y}{B_2L_t r_{0t}}, \mu_{1t} = \frac{Y}{B_1L_t r_{0t}} \end{aligned}$$

To replace $\dot{\mu}_{1t}, \mu_{1t}$ in equation (10) with $\dot{\mu}_{1t} = -\frac{\dot{r}_{0t}}{r_{0t}}\mu_{1t}, \mu_{1t} = \frac{Y}{B_1L_t r_{0t}}$, and $\dot{\mu}_{2t}, \mu_{2t}$ in equation (11) with $\dot{\mu}_{2t} = -\frac{\dot{r}_{0t}}{r_{0t}}\mu_{2t}, \mu_{2t} = \frac{Y}{B_2L_t r_{0t}}$, we get:

$$\begin{aligned} -\frac{\dot{r}_{0t}}{r_{0t}}\frac{Y}{B_1L_t r_{0t}} &= (\rho + \delta_1)\frac{Y}{B_1L_t r_{0t}} - \frac{\beta}{E_t} \Rightarrow -\dot{r}_{0t} = \left((\rho + \delta_1)\frac{Y}{B_1L_t r_{0t}} - \frac{\beta}{E_t}\right)\frac{B_1L_t r_{0t}}{Y} r_{0t} \\ -\frac{\dot{r}_{0t}}{r_{0t}}\frac{Y}{B_2L_t r_{0t}} &= (\delta_2 + \rho)\frac{Y}{B_2L_t r_{0t}} - \frac{\alpha}{R_t} \Rightarrow -\dot{r}_{0t} = \left((\rho + \delta_2)\frac{Y}{B_2L_t r_{0t}} - \frac{\alpha}{R_t}\right)\frac{B_2L_t r_{0t}}{Y} r_{0t} \end{aligned}$$

At an equilibrium point, $\dot{r}_{0t}=0, \dot{E}_t=0$ and \dot{R}_t so we have:

$$(\rho + \delta_1)\frac{Y}{B_1L_t r_{0t}} - \frac{\beta}{E_t} = 0 \Rightarrow (\rho + \delta_1)\frac{Y}{B_1L_t r_{0t}} = \frac{\beta}{E_t} \quad (12)$$

$$(\rho + \delta_2)\frac{Y}{B_2L_t r_{0t}} - \frac{\alpha}{R_t} = 0 \Rightarrow (\rho + \delta_2)\frac{Y}{B_2L_t r_{0t}} = \frac{\alpha}{R_t} \quad (13)$$

$$\delta_1 E_t = B_1 L_t r_{1t} \quad (14)$$

$$\delta_2 R_t = B_2 L_t (1 - r_{0t} - r_{1t}) \quad (15)$$

From equation (12) and (14), we have:

$$(\rho + \delta_1)\frac{Y r_{1t}}{r_{0t}} = \beta \delta_1 \quad (16)$$

From equation (13) and (15), we have:

$$(\rho + \delta_2)\frac{Y(1 - r_{0t} - r_{1t})}{r_{0t}} = \alpha \delta_2 \quad (17)$$

From equation (16) and (17), we get:

$$r_{0t} = \frac{(\rho + \delta_1)(\rho + \delta_2)\gamma}{(\rho + \delta_1)\alpha\delta_2 + (\rho + \delta_2)\beta\delta_1 + (\rho + \delta_2)(\rho + \delta_1)\gamma}$$

$$r_{1t} = \frac{(\rho + \delta_2)\beta\delta_1}{(\rho + \delta_1)\alpha\delta_2 + (\rho + \delta_2)\beta\delta_1 + (\rho + \delta_2)(\rho + \delta_1)\gamma}$$

$$1 - r_{0t} - r_{1t} = \frac{(\rho + \delta_1)\alpha\delta_2}{(\rho + \delta_1)\alpha\delta_2 + (\rho + \delta_2)\beta\delta_1 + (\rho + \delta_2)(\rho + \delta_1)\gamma}$$

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