CONTRIBUTION OF P$_{11}$ (1440) RESONANCE TO THE d(e, e′ π$^+$)nn REACTION INCLUDING NN FINAL STATE INTERACTION

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Abstract

The effect of P$_{11}$(1440) (Roper) resonance on the response functions for the positivelly charged pion electroproduction from the deuteron is studied using the well known spectator model. The amplitude for the elementary reaction $\gamma^* N \rightarrow \pi^+ N$ is taken from MAID2003 model. The effect of the *NN* interaction subsystem in the final state interaction (FSI) is included. Two values for the four momentum transferred Q^2 are used. A small effect of the Roper resonance on the structure functions is found.

Introduction

The P$_{11}$(1440) (Roper resonance) is of particular interest among the low-lying nucleon excitations, not only because of its relatively low mass, but primarily bcause of the rather peculiar behavior of the scattering and electro-excitation amplitudes. This clearly indicates that the structure of the resonance cannot be explained by a simple excitation of the quark core (like most of the other low-lyin states).

The mesons, in particular the pion and the σ-meson, play an important role, yet the question remains whether it is possible to explain the Roper solely in terms of the quark and meson degrees of freedom or exotic degrees of freedom have to be incorporated like the explicit gluons [1–3].

The P$_{11}$(1440) (Roper) resonance [4] decay into πN with ratio (60-70) or into ηN with ratio (40-30) is the lowest positive-parity N* state. It is visible only indirectly in partial-wave analyses of $\pi N \rightarrow \pi N$ and $\pi N \rightarrow \pi \pi N$ scattering as a shoulder around 1440 MeV with a large width. The Roper is buried underneath the Born backgrounds and merges with the tails of other neighboring resonances (in particular the P$_{33}$(1232), D$_{13}$(1520), and S$_{11}$(1535)), and thus can not be resolved from the W-dependence of the cross-section alone. Furthermore, the methods by which the masses and widths of the Roper have been determined differ significantly: from πN scattering, a Breit-Wigner mass of 1470 MeV and width of 350 MeV is extracted, while a speed-plot analysis yields 1375 MeV and 180 MeV, respectively [5]. In addition, due to its high inelasticity, the Roper resonance has a very typical behavior of ImTπN and exhibits multiple T-matrix poles in the complex energy plane on auxiliary Riemann sheets. In privious paper [6] the effect of P$_{11}$(1440) (Roper) on the semi-exclusive structure functions of the incoherent *π^0- meson* electroproduction off the deuteron is studied at 0.01 and 0.1 GeV^2 four momentum transfer and different values of the incident virtual photon lab energy k_0^{lab} .

In this work the effect of P$_{11}$(1440) is studied for the *π$^+$- meson* electroproduction off the deuteron.

The present paper is organized as follows; the formalism of π^+- *meson* electroproduction off the deuteron with the final state interaction is briefly given in section 2. The results are summarized and some discussion is presented in section 3. At the end the conclusion is presented.

2- π^+- *meson* electroproduction off the deuteron

The quasifree positive pion electroproduction from the deuteron can be represented according to the next Eq.

$$\gamma^*(K) + d(p_d) \rightarrow \pi^+(q) + n(p_1) + n(p_2) \tag{1}$$

Where $K = (k_0, \vec{k})$, $p_d = (E_d, \vec{d})$, $q = (\omega, \vec{q})$ and $p_{1/2} = (E_{1/2}, \vec{p}_{1/2})$ denote the four-momenta of the incoming virtual photon, the initial deuteron, the outgoing π^+- *mesons* and the two outgoing nucleons, respectively and the energies are given by $E_d = \sqrt{M_d + \vec{d}^2}$, $E_{1/2} = \sqrt{M^2 + \vec{p}_{1/2}^2}$ and $\omega = \sqrt{m_\pi^2 + \vec{q}^2}$.

Reaction (1) is considered in γ^*d c.m. system where the z-axis is along the momentum of the incoming virtual photon $\vec{k}$, the y-axis parallel to $\vec{p}_e \times \vec{p}_e$ and the x-axis such as to form a right handed system. Therefore, the outgoing pion is described by the spherical angles Θ_π and ϕ_π see fig. (1).

According to pervious work [6-9], the four structure functions R_α with $\alpha \in$ {L, T, LT, TT} are given in terms of the T-matrix elements of the π^+- mesons electroproduction current operator $J\gamma\pi_{,\mu}(0)$ according to

$$T_{sm.\mu m_d} = -^{(-)}\langle \vec{p}_1 \vec{p}_2 \, sm_s, \vec{p}_\pi | J_{\gamma\pi.\mu}(0) | \vec{p}_d \, 1m_d \rangle , \tag{2}$$

where s and m_s denote the total spin of the outgoing nucleons and its projection on the z-axis, respectively, μ the polarization of the virtual photon, and m_d the deuteron spin projection.

Introducing a partial wave decomposition of the final states, one finds

$$T_{sm_s\mu m_d}(W, Q^2, p_\pi, \Omega_\pi, \Omega_p) = e^{i(\mu+m_d-m_s)\phi_\pi} t_{sm_s\mu m_d}(W, Q^2, p_\pi, \theta_\pi, \theta_p, \phi_{p\pi}) \tag{3}$$
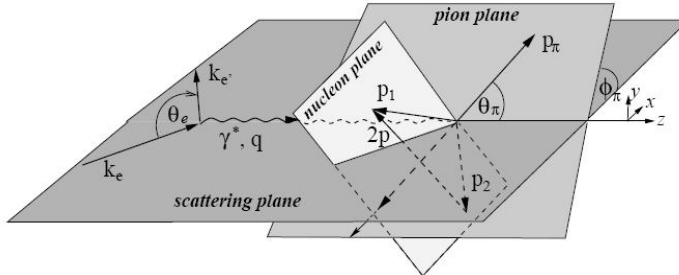

Figure. 1: Kinematics of single pion electroproduction on the deuteron in the γ*d cm. system.

where the small *t-matrix* depends besides W, Q^2 and p$_\pi$ only on θ_π, θp, and the relative azimuthal angle $\phi_r = \phi_p - \phi_\pi$[6] .

In the present work the elementary one-body pion production current of MAID-2003 as e.m. current [10] which has been developed for nuclear applications for photon energies up to 2 GeV. It contains Born terms, nucleon resonances P$_{33}$(1232), D$_{13}$(1520), S$_{11}$(1535), S$_{31}$(1620), S$_{11}$(1650), D$_{15}$(1675), F$_{15}$(1680), D$_{33}$(1700), P$_{13}$(1720), F$_{35}$(1905), P$_{31}$(1910), F$_{37}$(1950) and vector meson exchange is include.

The final state interaction, FSI, of the rescattering contributions in the final NN- subsystems is considered, Thus as in [9] we split the T -matrix into the impulse approximation (IA) T^{IA}, where final state interaction effects are neglected, and the rescattering contribution T^{NN} of the two-body NN- subsystem, respectively,

$$T_{sm_s\mu m_d} = T^{IA}_{sm_s\mu m_d} + T^{NN}_{sm_s\mu m_d}$$

(4)

For the IA contribution, where the final state is described by a plane wave, antisymmetrized with respect to the two outgoing nucleons, one has:-

$$T^{IA}_{sm_s\mu m_d} = \langle \vec{p}\, sm_s, \vec{p}_\pi \,|\, \big[j_{\gamma\pi,\mu}(1) + j_{\gamma\pi,\mu}(2)\big] \,|\, 1\, m_d \rangle$$
$$= \sqrt{2} \sum_{m'_s} \left(\langle sm_s \,|\, \langle \vec{p}_1 | j_{\gamma\pi,\mu}(W_{\gamma N_1}, Q^2)|\vec{p}_d - \vec{p}_2 \rangle \phi_{m'_s m_d}(\tfrac{1}{2}\vec{p}_d - \vec{p}_2)| 1\, m'_s \rangle - (1 \leftrightarrow 2) \right),$$

(5)

where $j_{\gamma\pi,\mu}$ denotes the elementary pion photoproduction operator of the MAID-2003 model, $W\gamma^*N_1$ the invariant energy of the γ^*N_1 system, $\vec{p}_{1/2} = (\vec{q} + \vec{p}_d - \vec{p}_\pi)/2 \pm \vec{p}$. Furthermore, Φ_{msmd} is related to the internal deuteron wave function in momentum space by

$$\langle \vec{p}, 1m_s | 1m_s \rangle^{(d)} = \phi_{m_s m_d}(\vec{p}) = \sum_{L=0,2} \sum_{m_L} i^L (Lm_L\, 1m_s | 1m_d) u_L(p) Y_{Lm_L}(\hat{p})$$

(6)

The *NN* rescattering contributions have a similar structure

$$T^{NN}_{sm_s\mu m_d} = \langle \vec{p}\, sm_s, \vec{p}_\pi \,|\, T_{NN} G_{NN} \big[j_{\gamma\pi,\mu}(W_{\gamma N_1}, Q^2) + j_{\gamma\pi,\mu}(W_{\gamma N_2}, Q^2)\big]\| 1\, m_d \rangle ,$$

(7)

where T^{NN} denote the NN scattering matrices and G$_{NN}$ the corresponding free two-body propagators.

Then the responce functions *given* by

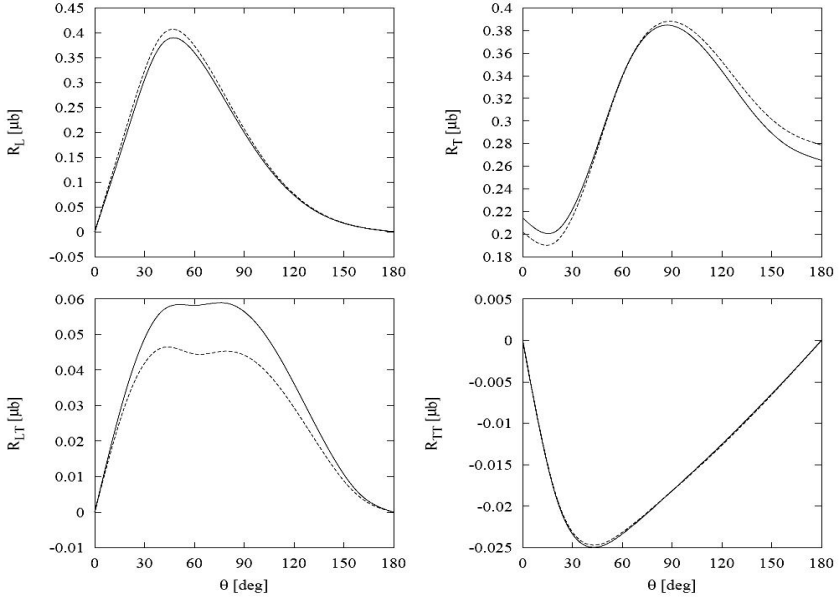$$R_L = w_{00}, \quad R_T = w_{11}, \quad R_{LT} = -\sqrt{2}\,\Re e\, w_{10}, \quad R_{TT} = w_{1-1}$$

(8)

where the reduced hadronic tensor

$$w_{\lambda'\lambda} = \sum_{smmd} \int d\,\Omega_p \,(T_{sms\mu md}(W, Q^2, p_\pi, \Omega_\pi, \Omega_p))^* (T_{sms\mu md}$$
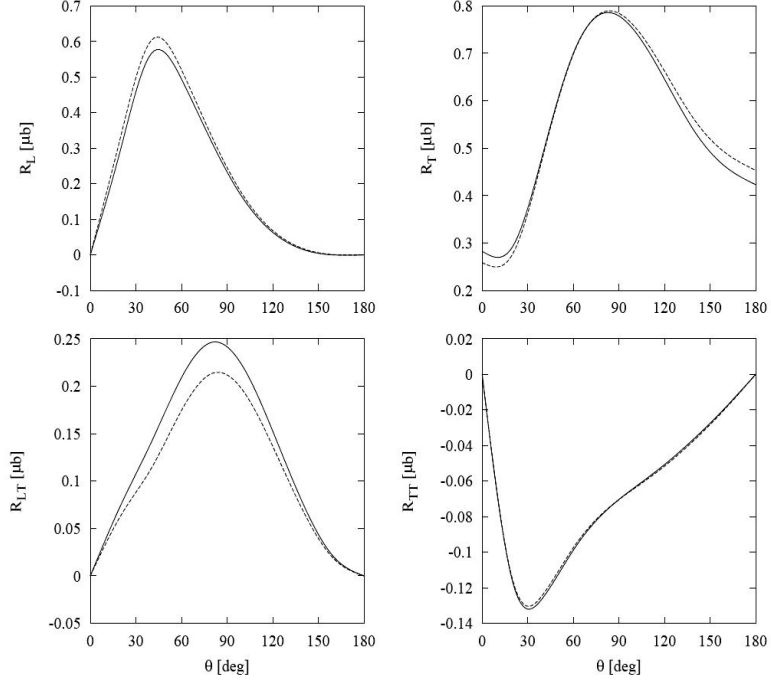$$(W, Q^2, p_\pi, \Omega_\pi, \Omega_p)$$

(9)

Results and discussion

In this section the numerical results for the response functions of possitive pion electroproduction from the deuteron with FSI of NN- subsystems are presented and discussed. As already mentioned, the realistic MAID-2003 model [10] has been used for the evaluation of the elementary pion electroproduction operator on the free nucleon. The electromagnetic production amplitude is parameterized in term of CGLN amplitudes given as numerical tables in the π^+ nucleon c.m. frame. This amplitude had to be generalized to an arbitrary frame of reference in order to be incorporated into the reaction on the deuteron. This was achieved by constructing from the MAID-2003 model Lorantz invariant amplitudes. This generalized elementary production operator was then used to evaluate π^+ electroproduction off the deuteron. The numerical evaluation is based on Gauss integration for the calculation of the matrix element of the MAID operator using for the deuteron wave function an analytical parameterization of the S- and D-waves of the Bonn potential in momentum space [13].

In figures 2-5 the angular distribution for the four structure functions at different values for the four momentum transfer (K^2) and the virtual photon lab energy $\left(k_0^{lab}\right)$ are presented. The dotted lines indicate the situation when the $P_{11}(1440)$ resonance contribution is eliminated from the elementary process and the continues ones are when this contribution is considered.

Figure 2: **Angular dependence of the four responce functions of** *d(e, e′ π⁺)nn* **at** k_0^{lab} **=250 MeV and squared four-momentum transfer** K^2 **=.01 (GeV/c)², full lines where the** P_{11} **(1440) is included and dotted lines where** P_{11} **(1440) is eliminated.**

Figure 3: Notation as in Fig.2 but for k_0^{lab} **=350 MeV.**

One easily notice that the main contributions come from the longitudinal (R_L) and transferee (R_T) structure functions, the interference functions R_{LT} and R_{TT} have to vanish at 0 and 180 degree due to helicity conservation.

Keeping the four momentum transfer constant ($K^2 = 0.1 \text{GeV}^2$) where the virtual photon lab energy takes different values (250 MeV for Fig 2, and 350 MeV for Fig. 3), one notice a small effect by including P$_{11}$(1440) resonance for R_{LT} and R_L where the effect is mostly neglected in case of R_{TT} , that maybe mean this transition is dominated by magnetic transition. . It is also noticeable that, increasing k_0^{lab} results in increasing the magnitude of the structure functions.

As a next, Figs (4 and 5) the four momentum transfer is keeping constant (K^2= 0.05 GeV^2) where the virtual photon lab energy takes two different values (k_0^{lab} 250 MeV for Fig 4 and 350 MeV for Fig. 5), The effect of P$_{11}$(1440) resonance still be very small, it result in small increase in the case of R_{LT} and also small decrease in case of R_L where nearly no effect for R_{TT}.

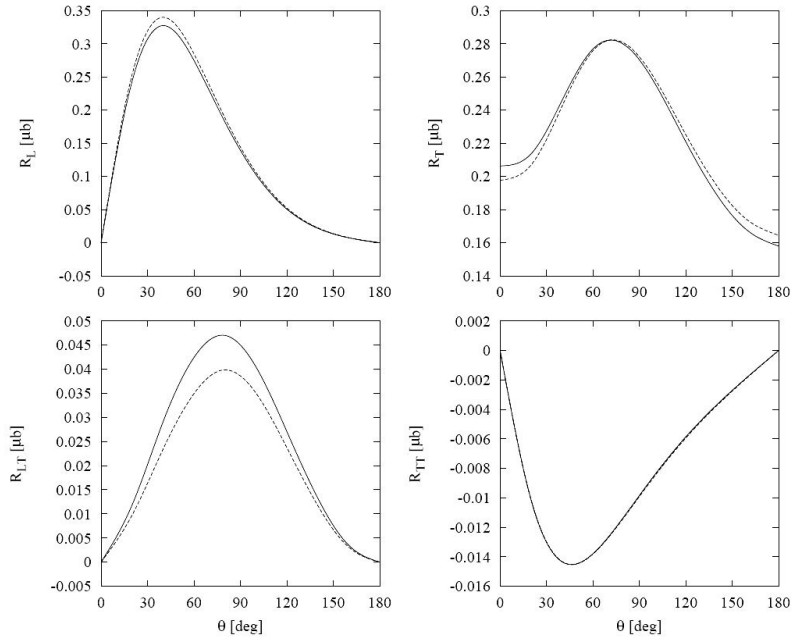Again, increasing k_0^{lab} results in increasing the magnitude of the structure functions.

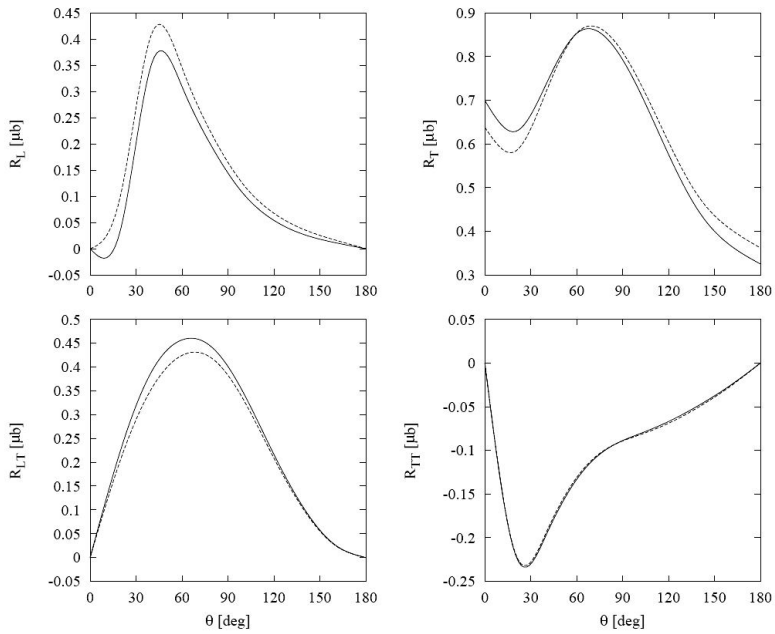**Figure 4: Notation as in Fig.2 but for K^2= 0.05 GeV^2.**

Figure 5: Notation as in Fig.3 but for K^2= 0.05 GeV^2.

4 Conclusion

A systematic study for the role of P$_{11}$(1440) resonance on the semi exclusive structure functions of the incoherent positive pion electroproduction off the deuteron is done. The NN interaction between the outgoing nucleons is included while the pion nucleon interaction is neglected. Since these structure functions depend on the squared four momentum transfer K^2, the invariant energy or equivalently the virtual photon laboratory energy k_0^{lab} and the outgoing pion angle θ_π in the final hadronic c.m. system, two values for K^2 and k_0^{lab} have been selected for the presentation of the results. The results show a small effect of P$_{11}$(1440) , this effect somewhat increase by increasing K^2.

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