Competition Effects in Presence of Dipole-dipole Interaction in Two Atom Systems: A Steady State Analysis

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Authors' contributions

This work was carried out in collaboration between all authors. Author SA has carried out a large part of the literature search, numerical calculations, some amount of discussions and interpretations and also typed the manuscript. Authors AV and PAL have more or less designed the study, contributed to major part of the analysis and wrote the draft manuscript and also carried out the corrections as the manuscript evolved. All authors have read the final manuscript and agreed upon its content and approved it as a whole. It is a joint effort of all the three authors.

Original Research Article

ABSTRACT

The behaviour of a system of two two-level atoms, both identical as well as non-identical, under the influence of a continuously varying dipole-dipole coupling parameter for a wide range of other parameters such as the atomic level separations and the coupling strength of the external radiation field is explored. A detailed analysis of the behaviour of various level populations and some of the atomic coherences is presented. The influence of the competing effects between the Rabi field strength and the dipole-dipole interaction on the two photon absorption probability is explored in detail both for identical as well as non-identical atoms. There are significant variations in the behaviour of identical and non-identical atoms as a function of the dipole coupling strength, even for small amounts of non-identity of the atoms, which in this study is incorporated through the atomic level separations. Numerical results of one of the measures of entanglement, namely the concurrence are presented.

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1. INTRODUCTION

Cooperative effects [1], for example collective emission, are a general characteristic of macroscopic systems consisting of a large number of atoms confined in a region of space much smaller than an optical wavelength. When treating a system of many atoms, the net force on any single atom is computed as a sum of interactive forces with all other atoms, one by one. Coherent addition of all these inter-atomic interactions gives rise to cooperative phenomena. Therefore the model for collective, macroscopic behaviour of the entire system can be built in terms of a basic building block involving interaction between a pair of atoms. Two of the most common interactions of this kind are dipole-dipole and Van der Waal's. Dipole-dipole interaction exists between two atoms wherein one of them is in an excited level and the other is in one of the lower lying levels in its manifold and in addition transition between the excited and lower levels of each atom needs to be dipole allowed. In such a case, the two atoms non-radiatively exchange energy leading to an effective interaction potential. Though Van der Waal's interaction is present between two atoms in identical states, it can be understood as a non-radiative energy exchange between the spin states of the atoms, leading to a mathematical frame work almost identical to that of dipole-dipole interaction. While a fully quantum mechanical picture describes the energy exchange phenomena more effectively, such an approach becomes cumbersome as the total number of atoms increases. With this in mind, we present a detailed study using semi classical approach wherein the interaction is represented by a coupling factor between relevant atomic states. In this particular communication, we concentrate on dipole-dipole interaction between two identical as well as two non-identical atoms, and present results of the effect of the dipole-dipole interaction on the behaviour of the atomic system.

A large amount of work exists that study cooperative effects among few atoms, resulting from pairwise dipole-dipole interactions [1-10], each showing one of the several outcomes of the dipole-dipole interactions. For example, new resonance fluorescence peaks that arise due to dipole-dipole interactions [11], suppressions of existing fluorescence peaks [12], modifications of transition rates and level shifts, thus giving rise to significant changes in the multiple jump dynamics [13] have been reported. More recently, entanglement between the atom pair arising due to dipole-dipole interactions [14] and their subsequent evolution [15] is studied. Dipole - dipole interaction between Rydberg atoms, which are stronger due to the large dipole moments [16-18], application of this to quantum computation schemes [19], have also generated much interest. There exist several other physical phenomena which have the same dynamics as that of two two-level atoms, such as modes in ring cavities, quantum dots, excitons in plasmonic waveguides etc. Several of them also show similar coupling mechanisms where a resonant energy transfer between any two entities exists. All of them can be solved with the same mathematical formalism shown here and will probably show similar behaviour as well, under the appropriate limits. Despite the large amount of work that already exists in literature, it can be stated that an understanding of the dipole-dipole interaction effects is far from complete.

In one of the early studies carried out by Varada and Agarwal [20], the presence of dipole-dipole interaction induced two-photon resonance for the case of non-identical atoms is reported. Part of this present work is an extension of this work, at first confirming these results and then extending to newer regions which were unexplored in the previous work. In particular, the dipole interaction induced two-photon resonance in a wider parameter regime is seen to exhibit anomalous behaviour. It increases at first up to a certain value of the dipole coupling strength and for further increase in this parameter, it begins to decrease. It would be of interest to explore the signature of this anomalous behaviour on the entanglement

characteristics of this system. The work in this paper addresses these questions in particular.

The organization of this paper is as follows. In Sec. 2, we present the mathematical formulation of the problem. Master equation method [21] for the evolution of the density operator, in the semi-classical approximation, is employed in the present study. The resulting density matrix equations after the usual rotating wave approximation are solved numerically for an extensive parameter regime. In section 3, numerical results of various quantities are presented for a wide choice of parameters, both for identical as well as non-identical atoms. It is seen that a small deviation from identity of the atoms gives rise to significant changes in the behaviour of the system.

2. THE MODEL

We consider two isolated two level atoms fixed in position with a separation r_{12}, driven by a single mode resonant laser field, of frequency ω_L(wavelength λ), which is nearly resonant with the transition frequency of one of the atoms, in the region $r_{12} \ll \lambda$. Here$|g_i\rangle$ and $|e_i\rangle$ are respectively the ground and excited states of the ith atom ($i = 1,2$). The interaction with the field induces dipole moment in the atoms, which in turn interact with each other via photon exchange. In our model it is assumed that the external field is propagating perpendicular to the inter-atomic axis so that the interaction of field with the atoms is purely symmetric.

Several theoretical approaches can be used to treat the system of two atoms interacting with the radiation field. One of the more preferred among these is the master equation approach, which enables one to treat the evolution of the atom plus field system entirely in terms of the atomic operators. The master equation which governs the dynamics of the density operator ρ of the two atom system is given by

$$\frac{\partial \rho}{\partial t} = -i \sum_{i=1}^{2} \Delta_i [S_i^z, \rho] - i \sum_{i \neq j=1}^{2} \Omega_{ij} [S_i^+ S_j^-, \rho] + i \frac{\Omega_0}{2} \sum_{i=1}^{2} [S_i^+ + S_i^-, \rho] - \mathcal{L}\rho \tag{2.1}$$

where the last term

$$\mathcal{L}\rho = \frac{1}{2} \sum_{i,j=1}^{2} \Gamma_{ij} (\rho S_i^+ S_j^- + S_i^+ S_j^- \rho - 2S_j^- \rho S_i^+)$$

represents the decay of the atomic system. Here $S_i^+ = |e_i\rangle\langle g_i|$ and $S_i^- = |g_i\rangle\langle e_i|$ are the raising and lowering operators, $S_i^z = \frac{1}{2}(|e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|)$ is the energy operator of the ith atom and $\Gamma_{ij} = \Gamma$ are the spontaneous decay rates.

The parameters Ω_{ij} and $\Gamma_{ij}(i \neq j)$ which describe the dipole-dipole interaction and the collective damping respectively are both functions of the distance between the atoms and are given by

$$\Omega_{ij} = \frac{3\Gamma}{4} \left[-(1 - \cos^2\theta) \frac{\cos(kr_{ij})}{kr_{ij}} + (1 - 3\cos^2\theta) \left\{ \frac{\sin(kr_{ij})}{(kr_{ij})^2} + \frac{\cos(kr_{ij})}{(kr_{ij})^3} \right\} \right] \tag{2.2}$$

and the decay term

$$\Gamma_{ij} = \frac{3\Gamma}{2}\left[(1-\cos^2\theta)\frac{\sin(kr_{ij})}{kr_{ij}} + (1-3\cos^2\theta)\left\{\frac{\cos(kr_{ij})}{(kr_{ij})^2} - \frac{\sin(kr_{ij})}{(kr_{ij})^3}\right\}\right] \tag{2.3}$$

where 'θ' is the angle between the dipole moment $\vec{\mu}$ and the interatomic separation $\vec{r}_{ij}$. Here, we assume, with no loss of generality, that the atomic dipole moments are parallel to each other and are polarized in a direction perpendicular to the inter-atomic axis, i.e., $\vec{\mu}_1=\vec{\mu}_2=\vec{\mu}$ and $\vec{\mu} \perp \vec{r}_{ij}$.

The wave number $k = \frac{\omega_0}{c}$, where $\omega_0 = \frac{\omega_1+\omega_2}{2}$, ω_1 is the atomic transition frequency of atom1 and ω_2 is the atomic transition frequency of atom2.

The density operator of the two-atom system can be represented in a complete set of basis states spanned by four product states, which is usually referred to as the standard basis, the elements of which are defined as

$$|1\rangle = |g_1\rangle \otimes |g_2\rangle$$
$$|2\rangle = |e_1\rangle \otimes |g_2\rangle$$
$$|3\rangle = |g_1\rangle \otimes |e_2\rangle$$
$$|4\rangle = |e_1\rangle \otimes |e_2\rangle \tag{2.4}$$

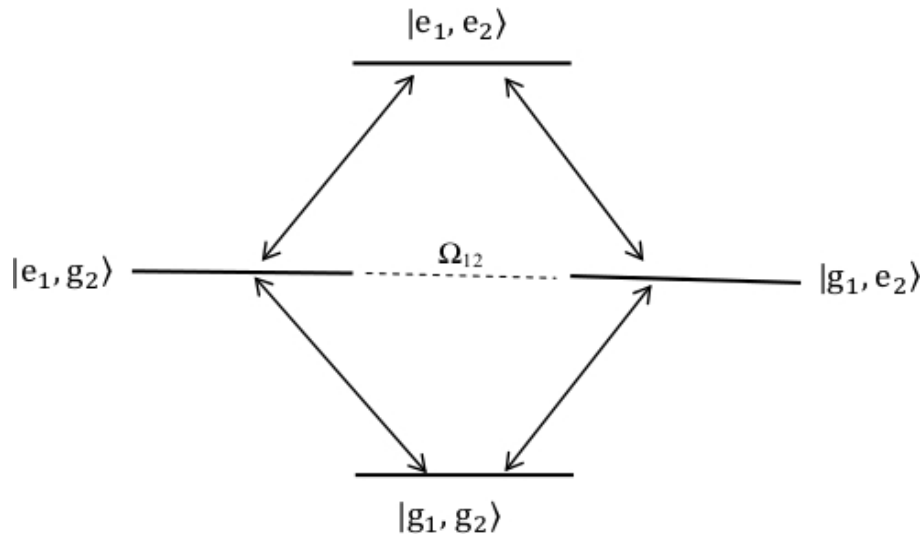A schematic of this level scheme is shown in Fig. 1.

Fig. 1. Schematic energy level diagram of two identical two-level atoms as an equivalent four-level system in product basis

However, since the dipole-dipole interaction Ω_{12} couples the two atoms, when describing the evolution of such a system, the standard basis is usually not the most convenient basis to work with. In this case, it is more convenient to include the dipole-dipole interaction into the Hamiltonian and re-diagonalize it, giving rise to a different set of basis states, which are termed as the collective states of the two-atom system, defined by

$$|g\rangle = |g_1\rangle \otimes |g_2\rangle$$
$$|s\rangle = \frac{1}{\sqrt{2}}[|e_1\rangle \otimes |g_2\rangle + |g_1\rangle \otimes |e_2\rangle]$$
$$|a\rangle = \frac{1}{\sqrt{2}}[|e_1\rangle \otimes |g_2\rangle - |g_1\rangle \otimes |e_2\rangle]$$
$$|e\rangle = |e_1\rangle \otimes |e_2\rangle \tag{2.5}$$

Unlike the standard basis states, the collective basis contains two intermediate states, $|s\rangle$ and $|a\rangle$ that are linear, symmetric and anti-symmetric super positions of the product states respectively, as shown in Fig. 2. Their energies depend on the dipole-dipole interaction and these states go through a large energy shift even for a small inter-atomic separation.
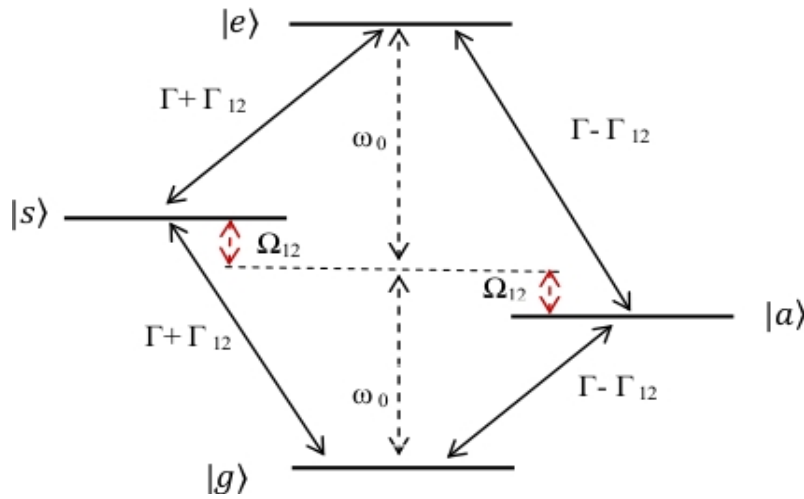

Fig. 2. Schematic energy level diagram of two identical two-level atoms as an equivalent four-level system in collective basis

In the rotating wave approximation (RWA), the density matrix elements transform as

$$\rho_{gg} = \tilde{\rho}_{gg} \; ; \rho_{ga} = \tilde{\rho}_{ga} \, e^{-i\Omega_0 t} \; ; \rho_{gs} = \tilde{\rho}_{gs} \, e^{-i\Omega_0 t} \; ; \rho_{ge} = \tilde{\rho}_{ge} \, e^{-2i\Omega_0 t}$$
$$\rho_{ag} = \tilde{\rho}_{ag} \, e^{i\Omega_0 t} \; ; \rho_{aa} = \tilde{\rho}_{aa} \; ; \rho_{as} = \tilde{\rho}_{as} \; ; \rho_{ae} = \tilde{\rho}_{ae} \, e^{-i\Omega_0 t}$$
$$\rho_{sg} = \tilde{\rho}_{sg} \, e^{i\Omega_0 t} \; ; \rho_{sa} = \tilde{\rho}_{sa} \; ; \rho_{ss} = \tilde{\rho}_{ss} \; ; \rho_{se} = \tilde{\rho}_{se} \, e^{-i\Omega_0 t}$$
$$\rho_{eg} = \tilde{\rho}_{eg} \, e^{2i\Omega_0 t} \; ; \rho_{ea} = \tilde{\rho}_{ea} \, e^{i\Omega_0 t} \; ; \rho_{es} = \tilde{\rho}_{es} \, e^{i\Omega_0 t} \; ; \rho_{ee} = \tilde{\rho}_{ee}$$

The resulting equations of motion for the sixteen density matrix elements after RWA are obtained as follows:

$$\frac{\partial \tilde{\rho}_{ee}}{\partial t} = i\widetilde{\Omega}(\tilde{\rho}_{se} - \tilde{\rho}_{es}) - 2\Gamma\tilde{\rho}_{ee}$$

$$\frac{\partial \tilde{\rho}_{ss}}{\partial t} = i\widetilde{\Omega}\big(\tilde{\rho}_{es} - \tilde{\rho}_{se} + \tilde{\rho}_{gs} - \tilde{\rho}_{sg}\big) + \frac{i(\Delta_2 - \Delta_1)}{2}(\tilde{\rho}_{as} - \tilde{\rho}_{sa}) - (\Gamma + \Gamma_{12})(\tilde{\rho}_{ss} - \tilde{\rho}_{ee})$$

$$\frac{\partial \tilde{\rho}_{aa}}{\partial t} = \frac{i(\Delta_1 - \Delta_2)}{2}(\tilde{\rho}_{as} - \tilde{\rho}_{sa}) - (\Gamma - \Gamma_{12})(\tilde{\rho}_{aa} - \tilde{\rho}_{ee})$$

$$\frac{\partial \tilde{\rho}_{gs}}{\partial t} = -i\tilde{\Omega} + \left[\frac{i(\Delta_1 + \Delta_2)}{2} - \frac{(\Gamma - \Gamma_{12})}{2} + i\Omega_{12}\right]\tilde{\rho}_{gs} + \frac{i(\Delta_1 - \Delta_2)}{2}\tilde{\rho}_{ga}$$
$$+ i\tilde{\Omega}(2\tilde{\rho}_{ss} + \tilde{\rho}_{aa} + \tilde{\rho}_{ee} - \tilde{\rho}_{ge}) + (\Gamma + \Gamma_{12})\tilde{\rho}_{se}$$

$$\frac{\partial \tilde{\rho}_{ga}}{\partial t} = \left[\frac{i(\Delta_1 + \Delta_2)}{2} - \frac{(\Gamma - \Gamma_{12})}{2} - i\Omega_{12}\right]\tilde{\rho}_{ga} + \frac{i(\Delta_1 - \Delta_2)}{2}\tilde{\rho}_{gs} - (\Gamma - \Gamma_{12})\tilde{\rho}_{ae} + i\tilde{\Omega}\tilde{\rho}_{sa}$$

$$\frac{\partial \tilde{\rho}_{ge}}{\partial t} = [i(\Delta_1 + \Delta_2) - \Gamma]\tilde{\rho}_{ge} + i\tilde{\Omega}(\tilde{\rho}_{se} - \tilde{\rho}_{gs})$$

$$\frac{\partial \tilde{\rho}_{se}}{\partial t} = \left[\frac{i(\Delta_1 + \Delta_2)}{2} - \frac{(3\Gamma + \Gamma_{12})}{2} - i\Omega_{12}\right]\tilde{\rho}_{se} + \frac{i(\Delta_2 - \Delta_1)}{2}\tilde{\rho}_{ae} - i\tilde{\Omega}(\tilde{\rho}_{ss} - \tilde{\rho}_{ee} - \tilde{\rho}_{ge})$$

$$\frac{\partial \tilde{\rho}_{ae}}{\partial t} = \left[\frac{i(\Delta_1 + \Delta_2)}{2} - \frac{(3\Gamma - \Gamma_{12})}{2} + i\Omega_{12}\right]\tilde{\rho}_{ae} + \frac{i(\Delta_2 - \Delta_1)}{2}\tilde{\rho}_{se} - i\tilde{\Omega}\tilde{\rho}_{as}$$

$$\frac{\partial \tilde{\rho}_{as}}{\partial t} = \frac{i(\Delta_2 - \Delta_1)}{2}(\tilde{\rho}_{ss} - \tilde{\rho}_{aa}) - (\Gamma - 2i\Omega_{12})\tilde{\rho}_{as} - i\tilde{\Omega}(\tilde{\rho}_{ae} + \tilde{\rho}_{ag})$$

$$\frac{\partial \tilde{\rho}_{gg}}{\partial t} = -\frac{\partial \tilde{\rho}_{aa}}{\partial t} - \frac{\partial \tilde{\rho}_{ss}}{\partial t} - \frac{\partial \tilde{\rho}_{ee}}{\partial t}$$

(2.6)

The remaining equations can be obtained from the Hermitian properties of the density matrix elements, viz.,

$$\tilde{\rho}_{ij} = \tilde{\rho}^*_{ji}$$

In the above equations the atomic detuning $\Delta_i = \omega_i - \omega_L, i = 1,2$, where ω_i is the frequency of ith atom, ω_L is the frequency of applied field and the total two-atom detuning $\Delta = \Delta_1 + \Delta_2$. Here $\Gamma_1 = \Gamma_2 = \Gamma$ is the spontaneous decay rate of each atom and $\tilde{\Omega} = \frac{\Omega_0}{2}$ is the modified Rabi frequency. A detailed inspection of the equations of motion reveals that the symmetric state is super radiant with rate $\Gamma + \Gamma_{12}$, whereas the anti-symmetric state is sub-radiant with rate $\Gamma - \Gamma_{12}$. The slowly decaying state $|a\rangle$ can be populated through two different channels, viz., the spontaneous emission from the state $|e\rangle$ and the coherent interaction with the state $|s\rangle$. By using the completeness condition of the level populations $\tilde{\rho}_{gg} + \tilde{\rho}_{aa} + \tilde{\rho}_{ss} + \tilde{\rho}_{ee} = 1$, the 16 equations can be reduced to 15 equations which can be cast in the form

$$\frac{\partial \Psi}{\partial t} = M\Psi + \Phi$$

(2.7)

where M is a 15 ×15 coefficient matrix and Ψ,Φ are column vectors each of length 15 which are defined in the following:

$$\Psi = [\tilde{\rho}_{ga} \ \tilde{\rho}_{gs} \ \tilde{\rho}_{ge} \ \tilde{\rho}_{ag} \ \tilde{\rho}_{aa} \ \tilde{\rho}_{as} \ \tilde{\rho}_{ae} \ \tilde{\rho}_{sg} \ \tilde{\rho}_{sa} \ \tilde{\rho}_{ss} \ \tilde{\rho}_{se} \ \tilde{\rho}_{eg} \ \tilde{\rho}_{ea} \ \tilde{\rho}_{es} \ \tilde{\rho}_{ee}]^{T} \tag{2.8}$$

$$\Phi = [0 - i\tilde{\Omega} \ 0 \ 0 \ 0 \ 0 \ 0 \ i\tilde{\Omega} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^{T} \tag{2.9}$$

Depending on the information that is being sought, the equations can be solved either as a function of time, which will necessitate solution of 16 first order coupled differential equations, or in steady state. In particular, the steady state solution of the density matrix elements is obtained as:

$$\Psi_{ss} = \Psi(t \to \infty) = -M^{-1}\Phi \tag{2.10}$$

As the focus here is on the steady state behaviour of the two-atom system, we present numerical results of different quantities, both for identical as well as non-identical atoms, in the next section. In the equations of the density matrix elements the '~' above each of the elements denotes that the elements are in rotating frame. In the next section, we remove the '~' for brevity of notation with the understanding that all the elements are defined in the rotating frame.

3. RESULTS AND DISCUSSION

The steady state behaviour of level populations, various atomic coherences and concurrence are studied numerically for a wide parameter range, both for identical and non-identical atoms. In this section, some representative numerical results for typical values of the parameters are presented. Throughout the results presented here, all the parameters, namely the atomic detunings, the Rabi frequencies, the dipole-dipole interaction constant are all normalized with respect to the population decay rate Γ.

3.1 Identical Atoms

The steady state populations of different levels in the collective basis, namely ρ_{gg}, ρ_{ss}, ρ_{aa} and ρ_{ee} for the case of identical atoms are plotted in Fig. 3, as a function of the two-atom detuning , for a fixed value of the scaled Rabi frequency Ω_0 and for different values of the dipole coupling parameter Ω_{12}.

For the case of identical atoms, it is found that the singlet anti symmetric state $|a\rangle$ acts as a dark state or a trapping state. This is so because the atomic coherences between this state and the rest of the triplet states are always found to be equal to zero. In this case, it is observed that the height of the central peak steadily decreases with an increase in the dipole coupling strength. Some representative values of the parameters are chosen to demonstrate this feature in Fig. 3, in which each of the sub-figures corresponds to a different value of the inter-atomic spacing which in turn defines the dipole coupling strength. For example, the steady state level populations which are plotted in Fig. 3(a) - 3(d), show the behaviour for three values of $\frac{r}{\lambda} = \frac{1}{10}, \frac{1}{14}$ and $\frac{1}{18}$. The corresponding dipole coupling parameter for these values of inter-atomic spacing, as obtained from equation 2.2 is given by Ω_{12}= 2.6, 7.6 and 16.6.

For a non-zero dipole-dipole interaction, the two - atom system has resonances at = 0 and = $-2\Omega_{12}$. At = $-2\Omega_{12}$, only the ground and symmetric states [cf. Fig. 3(a), 3(c)] are significantly populated. Lack of a corresponding peak in ρ_{ee}, shown in Fig. 3(d), can be

understood from the fact that the alternate pathways for stepwise excitation $|g\rangle \rightarrow |s\rangle \rightarrow |e\rangle$ and $|g\rangle \rightarrow |a\rangle \rightarrow |e\rangle$ are both inhibited. This feature is generally referred to as dipole blockade. The central resonance (peak at $\Delta = 0$) in ρ_{ee} results from a simultaneous excitation of both atoms ($|g\rangle \rightarrow |e\rangle$) which is termed as the two-photon resonance [20]. The resonances in other intermediate state populations at $\Delta = 0$ can be interpreted as arising from subsequent decay from $|e\rangle$ to these states. With increasing Ω_{12}, the value of the resonant peak at $\Delta = 0$ identically decreases for all the excited states, i.e., inhibition of two photon resonance due to the dipole-dipole interaction. A careful inspection of the behaviour of populations reveals that the step-wise excitation is more inhibited than the direct excitation.
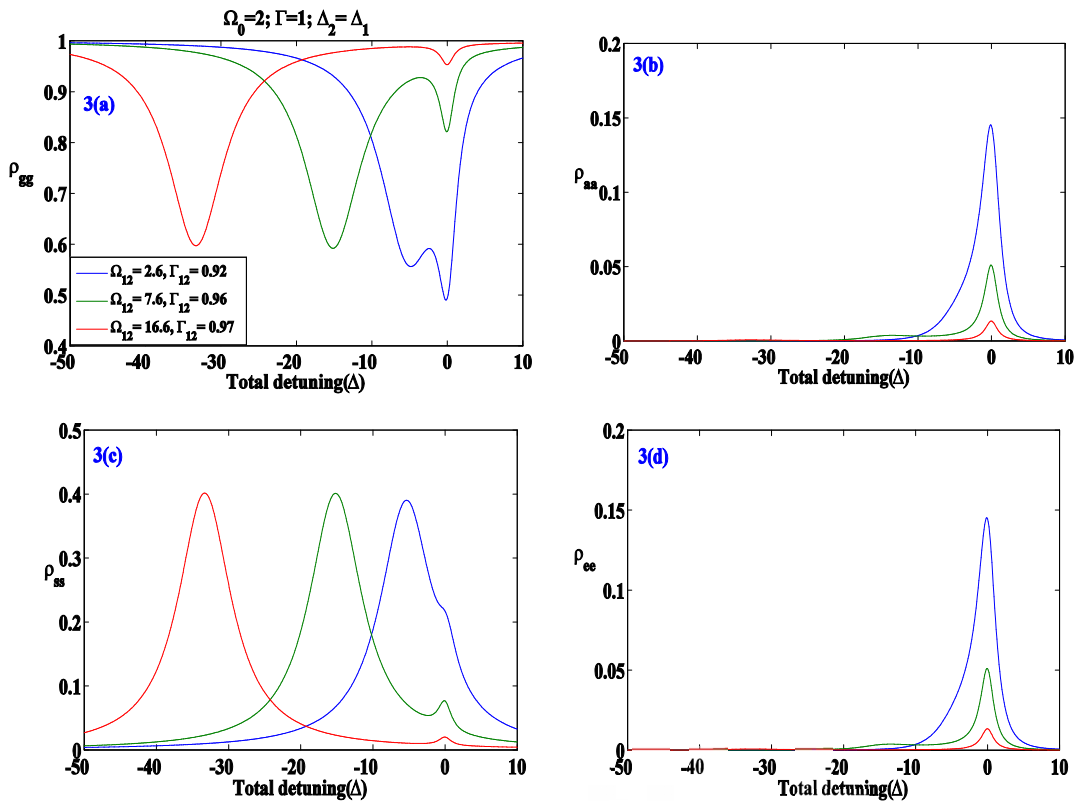

Fig. 3. Level populations for identical atoms $\rho_{gg}, \rho_{aa}, \rho_{ss}$ and ρ_{ee} corresponding to inter - atomic distances of $r = \frac{\lambda}{10}, \frac{\lambda}{14}$ and $\frac{\lambda}{18}$

3.2 Non-identical Atoms

For the case of non-identical atoms, we assume different energy level separation between ground and excited states of the two atoms and hence $\Delta_1 \neq \Delta_2$. We also assume that the two decay constants Γ_1 and Γ_2 are close enough, that they can be effectively considered equal, denoted by Γ. This is not physically inconsistent since in comparison to values of Rabi frequencies and detunings, the value of decay constants of different atoms are much closer. For example, ^7Li and ^{85}Rb have decay rates 5.92 MHz and 5.98 MHz respectively while

their resonant wavelengths are 670 nm and 780 nm. In such case, without any loss of generality, the decay constants of both species can be replaced by an identical value of 5.9MHz. However, when excited by a single mode radiation field, the detunings of each of these atoms would be markedly different from each other. Without a need to choose specific atomic species, we assume a situation of $\Delta_2 = \Delta_1 + 10\Gamma$.

The steady state level populations for the case of non-identical atoms, as a function of the total detuning, are presented in fig. 4. There are three resonance dips in ρ_{gg} for this condition, each dip mirrored by a corresponding peak in ρ_{ss}. The level population ρ_{ee} also shows this mirroring with two side peaks of small amplitude, indicating a step wise excitation of both the atoms, which was very strongly blocked for the case of identical atoms.
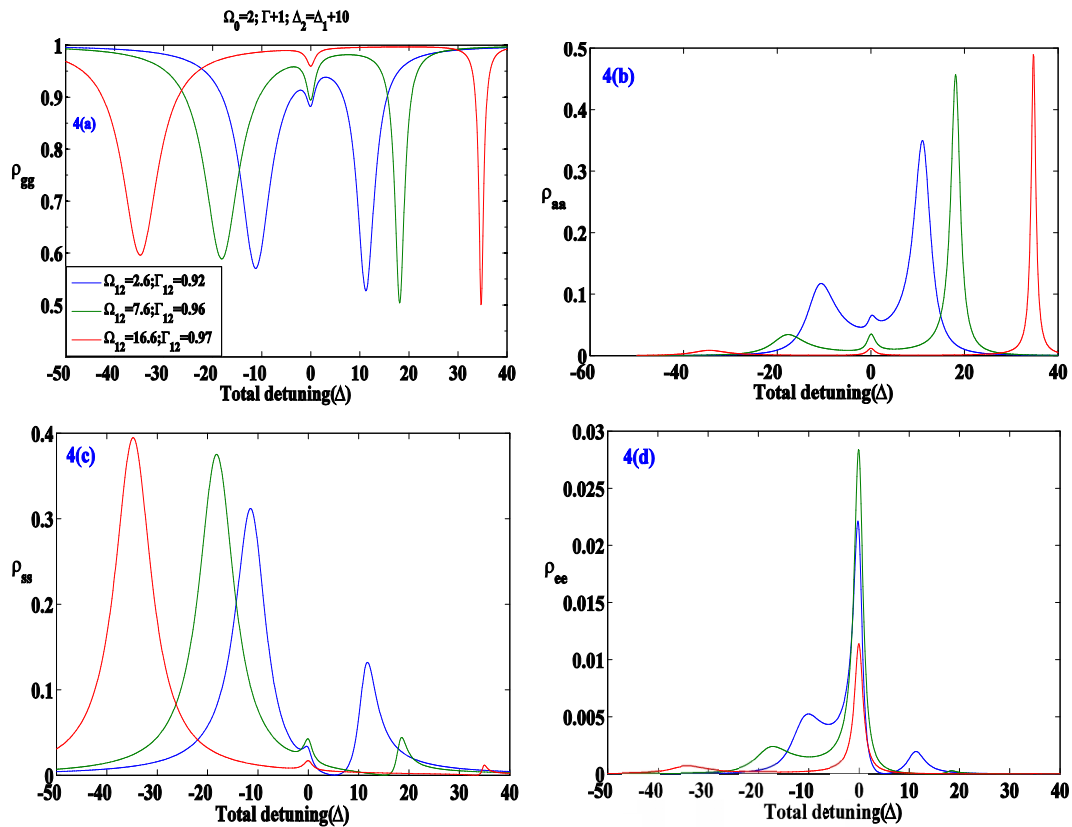

Fig. 4. Level populations for non-identical atoms ρ_{gg}, ρ_{aa} ρ_{ss} and ρ_{ee} corresponding to inter - atomic distances of $r = \frac{\lambda}{10}, \frac{\lambda}{14}$ and $\frac{\lambda}{18}$

From an eigenvalue analysis (shown in Fig. 5), the positions of the side bands are given as

$$\Delta = 2\sqrt{\left|\Omega_{12}\right|^2 + \frac{\delta^2}{4}} \quad \text{and} \quad \Delta = -2\sqrt{\left|\Omega_{12}\right|^2 + \frac{\delta^2}{4}}, \quad \text{where } \delta = \Delta_2 - \Delta_1. \quad \text{The eigenvalues}$$

respectively are $\dfrac{\Delta}{2}, -\dfrac{\Delta}{2}, \sqrt{\left|\Omega_{12}\right|^2 + \dfrac{\delta^2}{4}}, -\sqrt{\left|\Omega_{12}\right|^2 + \dfrac{\delta^2}{4}}$. The positions of the side peaks

can be ascertained from the difference between the Ω_{12} dependent eigenvalues.
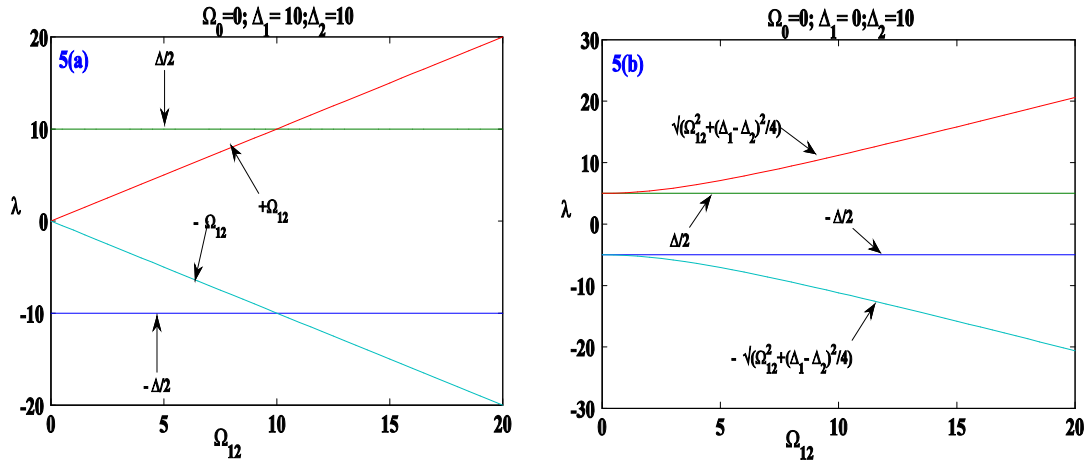

Fig. 5. Eigenvalue spectrum for (a) identical atoms (b) non-identical atoms

Unlike the behaviour exhibited in the case of identical atoms, for the non-identical atoms, one observes the presence of side bands in ρ_{ss}, ρ_{gg} as well as in ρ_{aa}. The corresponding side bands in ρ_{ee} are of small amplitude, indicating a reduced degree of inhibition of stepwise excitation in this case.

The central peak at $\Delta = 0$, which corresponds to simultaneous two-atom excitation, shows very interesting behaviour. As Ω_{12} is increased, the height of the peak initially increases and further increase in Ω_{12} gives rise to a decrease in the peak height. Varada and Agarwal [20] have calculated the probability for direct two-photon resonance, for non-identical atoms and have shown that this resonance is not present when $\Omega_{12} = 0$. It has to be noted that in our case the central peak in ρ_{ee} is a result of the combined effect of the population that can reach the state $\mid e\rangle$ through a stepwise excitation as well as direct excitation of the two atoms, which is obtained from a full density matrix calculation. Hence in our case, we notice the presence of the central peak even in the absence of Ω_{12}.

The reasons for the increase and subsequent decrease in the peak height, as the dipole-dipole interaction strength is continuously increased, are not obvious from the above analysis. However, looking at the eigenvalue evolution [cf. Fig. 5], one can say that Ω_{12} causes a shift of the energies of the levels $\mid s\rangle$ and $\mid a\rangle$. A critical value of Ω_{12}, say $\Omega_{12}(c)$, exists such that, at this dipole-dipole coupling strength, the shifted energies of $\mid s\rangle$ and $\mid a\rangle$ become equal to that as seen in the case of identical atoms. In other words, the non-identical nature of the atoms is compensated by the energy shifts due to this $\Omega_{12}(c)$. Further increase in the value of Ω_{12} beyond this critical value, will show a

decrease in the central peak height of ρ_{ee}, same as the feature seen in the case of identical atoms.

In order to obtain a parametric relation for this critical value $_{12}(c)$, computations were carried out for a wide range of parameters. From a careful analysis of the data thus obtained, it is inferred that $_{12}(c)$ is given by $\Delta_2 - \Delta_1$. The choice of the values of Ω_{12} in the figure is made such that this effect can be clearly demonstrated for the chosen values of the other parameters, in particular $\Delta_2 - \Delta_1 = 10\Gamma$. For example, one can clearly see the increase in the central peak height of ρ_{ee} [cf. Fig. 4(d)], as Ω_{12} is increased from 2.6 (blue online) to 7.6 (green online) (corresponding to the range below $_{12}(c)$). The third value 16.6 (red online) is chosen to be greater than $_{12}(c)$, where the decrease in the central peak height is clearly seen.

Further insight into the behaviour of this central peak can be obtained by looking at the atomic coherence ρ_{eg}. In the next Fig. 6 the imaginary part of the atomic coherence ρ_{eg} at the line-center ($\Delta = 0$) is plotted, as a function of kr for three values of Rabi field strength Ω_0. One notices that, in particular for higher values of the Rabi field strength, the pattern of increase, decrease and a second increase, as evident from the two peaks in Im (ρ_{eg}) is similar to the behaviour exhibited by the central peak of ρ_{ee}. The small differences in the detail here can be attributed to the fact that the central peak in ρ_{ee} is a result of all the pathways of excitation put together whereas the atomic coherence ρ_{eg} is a two-photon coherence.
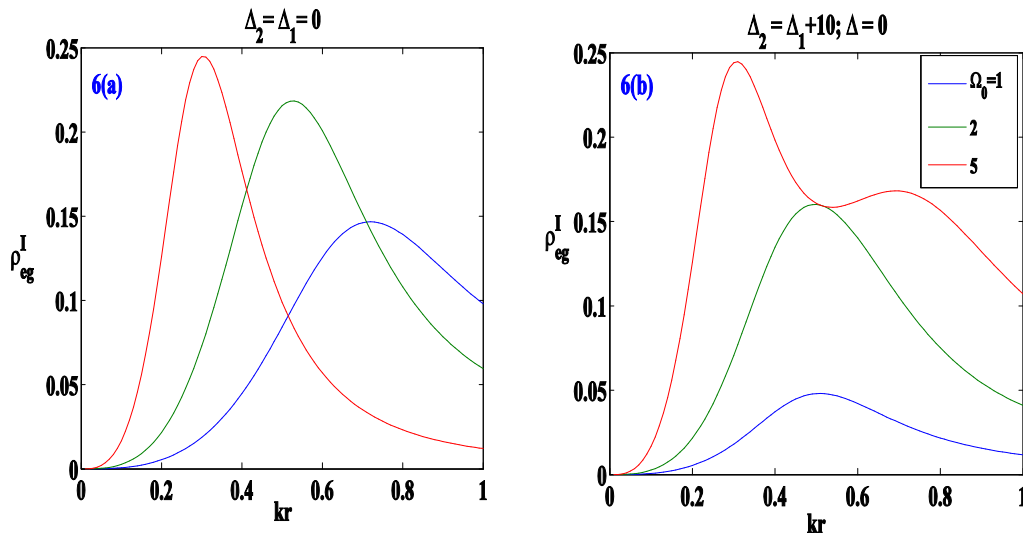

Fig. 6. The atomic coherence Im (ρ_{eg}) on resonance for (a) identical atoms (b) non-identical atoms

3.3 Concurrence

In this sub-section, we present results of concurrence, which is a measure of entanglement. The concurrence is calculated using the method elaborated in [22,19], for various values of kr and Ω_0 in the region of validity of dipole approximation $kr < 1$. Three dimensional plots

of the two-photon excited state population ρ_{ee} are presented in Fig. 7 and the corresponding concurrences are presented in Fig. 8.
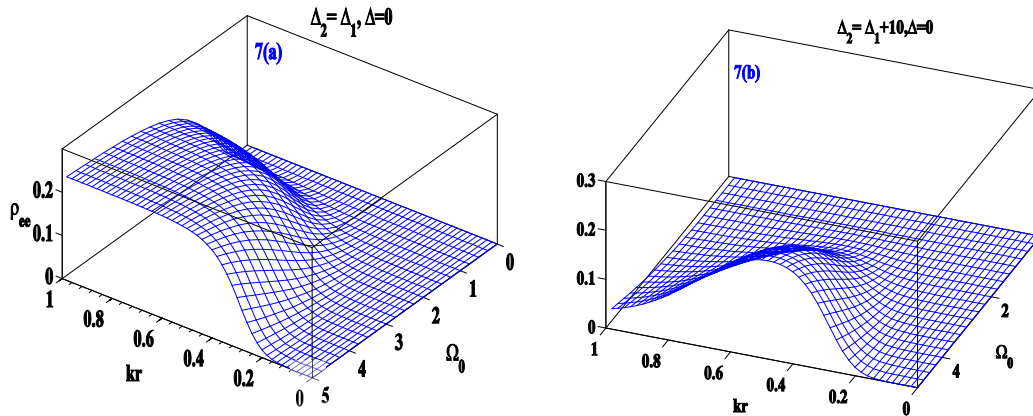

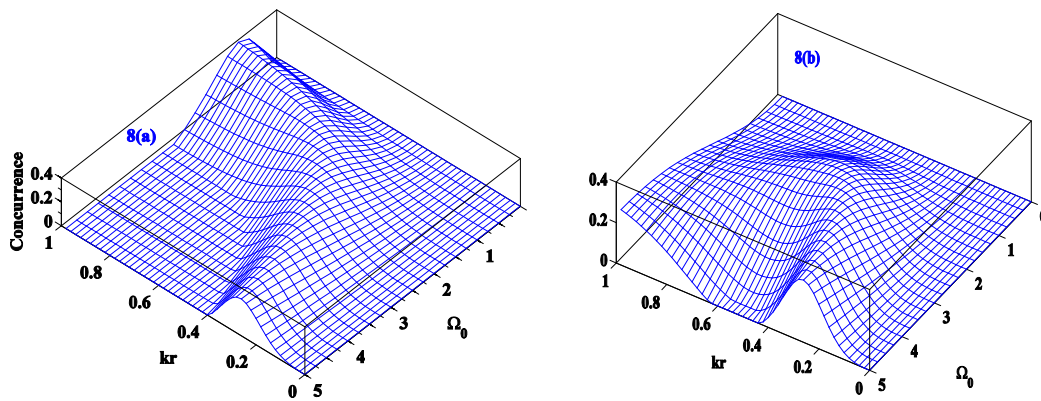**Fig. 7. Steady State ρ_{ee} for the case of (a) identical atoms (b) non-identical atoms**

Fig. 8. Steady State concurrence for the case of (a) identical atoms (b) non-identical atoms

In the case of identical atoms, for a given value of Ω_0, the concurrence peaks at a specific value of kr. For all other values of kr, concurrence is minimum and in addition is equal to zero whenever $\Omega_{12} < \Omega_0^2/2$. For non-identical atoms, the concurrence shows a more complex behaviour as is clear from figure 8 (b).

For moderate values of Ω_0, as kr is increased from 0 to 1, concurrence gradually increases. For higher values of Ω_0 (4 and above), there is a region of kr in which the concurrence goes to zero and remains zero, which can be interpreted as sudden death of entanglement. Further increase of kr for the same range of Ω_0 exhibits non - zero concurrence, which on the same token can be understood as entanglement re-birth.

In summary, a careful inspection of Fig. 7 and 8 clearly shows that the parameter zones where dipole-blockade is maximized, which is same as decrease in the line-center

population ρ_{ee}, are the same regions where concurrence is maximum. This is understandable because presence of dipole blockade is also an indicator of entanglement. From the results of Fig. 7, we can infer that the two-atom excitation depends upon two parameters, the Rabi coupling strength Ω_0 and the dipole-dipole interaction Ω_{12}. There is a competing effect with respect to how these two parameters affect simultaneous excitation of both atoms. The eigen value evolution with respect to Ω_{12} is linear in case of identical atoms and non-linear in case of non-identical atoms [cf. Fig. 5]. This appears surprising at first since for this particular case, we assume the same Ω_0 for both atoms even when they are non-identical. But the fact is that resonant two-atom excitation happens at $\Delta = 0 = \Delta_1 + \Delta_2$. This implies that effective coupling ratio Ω_0 / Δ_1 for one atom differs from the other Ω_0 / Δ_2, which leads to different couplings. This strongly suggests that each atom has a set of values for Ω_0, Ω_{12} and Δ at which the probability of excitation is maximum. However, this is different from the value at which probability for both atoms getting excited together is maximum.

The study presented here quantifies the relationship between dipole-blockade and entanglement. These quantitative connections between different atomic properties provide an idea of the parameter zones where this system can be exploited for the purpose of quantum information processing.

4. CONCLUSION

A system of two two-level atoms interacting with a single mode radiation field and interacting with each other through the dipole-dipole interaction is studied. Results of different level populations, the atomic coherence and the concurrence are presented for a wide variety of parameters:

a) The two photon resonance which appears in the presence of the dipole coupling strength shows an increase up to certain values of the coupling strength and beyond this critical value, it decreases thus exhibiting anomalous behaviour, only for the case of non-identical atoms. However, this anomalous behaviour is observed in the coherence $\square_\square$ both for identical as well as non-identical atoms.

b) The physical reasons for the non-monotonic behaviour of two-atom excitation in case of non-identical atoms require further investigation. However, it can be explained in a simple sense of being due to different set of parameters Ω_0, Ω_{12} and Δ at which each atom individually has maximum excitation probability, which in turn is different from the probability for simultaneous excitation. Varying any of the parameter will traverse over the probability curve showing a non-monotonic behaviour.

c) From a detailed analysis of the two-atom excited state population and the concurrence, a qualitative connection between the dipole-blockade and the entanglement is made.

d) To summarise, an attempt has been made to provide a range of parameter values within which the properties of the atomic system can be manipulated in a desired fashion for applications such as quantum information processing.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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