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Effects of relativity on dust ion acoustic shock waves containing degenerate electron-positron-ion dense plasma.

L. Nahar[1], M. S. Zobaer[2], N. Roy[3], and A. A. Mamun[4]

[1,4] *Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh.*
[2] *Lecturer of Physics, Bangladesh University of Textiles, Tejgaon, Dhaka, Bangladesh.*
[3] *Lecturer of Physics, United International University, Dhanmondi, Dhaka, Bangladesh.*

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Abstract

The nonlinear propagation of dust ion acoustic (DIA) waves in an unmagnetized collisionless degenerate dense plasma (containing degenerate electron, positron, and ion fluids) have been theoretically investigated. The Burgers' equation has been derived by employing the reductive perturbation method taking the effect of viscous force of the ion fluid into account. The stationary shock wave solution of Burgers' equation is obtained, and numerically analyzed in order to identify the basic properties of dust ion acoustic shock structures. It has been shown that the degenerate plasma under consideration supports compressive or rarefactive shock structures depending on some plasma parametric values. It has been also found that the effects of degenerate pressures of electrons, positrons, and ions significantly modify the basic features of the shock waves that are found to exist in such a degenerate plasma. The relevance of our results in astrophysical objects like white dwarfs and neutron stars, which are of scientific interest, is briefly discussed.

Keywords: Degenerate dense plasma, dust ion acoustic wave, Burgers' equation, shock wave, relativity.

1 Introduction

Dust is one of the omnipresent ingredients in our universe. The presence or dynamics of charged dust grains in a plasma not only modifies the existing plasma wave spectra, but also introduces a number of novel eigenmodes, such as the dust-ion-acoustic (DIA) waves, the dust-acoustic (DA) waves, the dust lattice waves, etc (1; 2). Shukla and Silin have first theoretically shown that due to the conservation of equilibrium charge density $n_{e_0}e + Z_d n_{d_0}e = n_{i_0}e$, and the strong inequality $n_{e_0} << n_{i_0}$ (where n_{i_0}, n_{e_0} and n_{d_0}, are the ion, electron and dust number density respectively, Z_d

[1] *Corresponding author: E-mail: l.nahar@rocketmail.com*

is the number of electrons residing into the dust grain surface and e is the magnitude of the charge of an electron) a dusty plasma (with negatively charged static dust) supports low-frequency DIA waves with phase speed much smaller (larger) than electron (ion) thermal speed (3). In our manuscript, we have considered a dusty plasma system containing electrons, positrons, ions, and negatively charged stationary dust particles which obeys the quasi-neutrality condition following the equation $n_{e_0}e + Z_d n_{d_0} e = n_{i_0}e + n_{p_0}e$ (where n_{e_0}, n_{p_0}, n_{i_0}, and $Z_d n_{d_0}$, are the number density of electrons, positrons, ions, and dusts respectively and Z_d is the number of electrons residing on the dust grain surface) under relativistic degenerate pressure.

The interest in plasma shock waves was first stimulated by the possibility of heating ions rapidly to thermonuclear temperatures. Under these conditions, the plasma and the shocks would necessarily be free from classical binary collisions, that is collisionless (4). However, the same collisionless shocks occur in some geophysical and astrophysical objects (compact objects like white dwarfs, neutron stars etc.) which is the subject matter of this work. Shock waves are formed due to the rapid compression of a plasma. The compression of such plasma results in energy dissipation which deals with viscous properties of systems with Coulomb potential (4; 5).

Of the nonlinear excitations, dust ion-acoustic (DIA) shock waves are of the most important and well-understood characteristics of plasma environments. Theoretical studies on main properties of these shock structures is carried out from 1961 (6). The method which is widely used to investigate the collective wave phenomenon in plasma is the so-called multi-scales perturbation method (7; 8). However, this method is based on approximation and used only for the small-amplitude treatment of plasma in a state away from thermodynamic equilibrium. Therefore, to obtain a good agreement with experiments, in this method, one needs to take higher-orders in perturbation modes in case of amplitudes of electrostatic structures. In recent years there have been many investigations on DIA shock as well as electrostatic shock waves in diverse plasma environments (9; 10; 11).

The electron-positron plasmas are thought to be generated naturally by pair production in high energy processes in the vicinity of several astrophysical objects as well as in laboratory plasmas experiments with a finite life time (12). Because of the long life time of the positrons, most of the astrophysical (13) and laboratory plasmas (14) become an admixture of electrons, positrons, and ions (15). It has also been shown that over a wide range of parameters, annihilation of electrons and positrons, which is analog of recombination in plasma composed of ions and electrons, is relatively unimportant in classical (16), as well as in dense quantum plasmas (17) to study the collective plasma oscillations. The ultra-dense degenerate electron-positron plasmas with ions are believed to be found in compact astrophysical bodies like neutron stars and the inner layers of white dwarfs (17; 18; 19) as well as in intense laser-matter interaction experiments (20). Therefore, it seems unimportant to study the influence of quantum effects on dense e-p-i plasmas. Several authors have theoretically investigated the collective effects in dense magnetized e-p-i quantum plasmas (21; 22) under the assumption of low-phase velocity (in comparison with electron/positron Fermi velocity) (23; 24; 25). In these studies, the authors focused on the lower order quantum corrections appearing in the well known classical modes.

Now-a-days, a number of authors have become interested to study the properties of matter under extreme conditions (26; 27; 28; 29). Recently, a number of theoretical investigations have also been made on the nonlinear propagation of electrostatic waves in degenerate quantum plasma by a number of authors (30; 31). However, these investigations are based on the electron equation of state valid for the non-relativistic limit. Some investigations have been made of the nonlinear propagation of electrostatic waves in a degenerate dense plasma based on the degenerate electron equation of state valid for ultra-relativistic limit (32; 33; 34; 35; 36). We are interested to study the dissipation relation of the dust ion-acoustic waves in a degenerate e-p-i plasma system where we added positrons for the rather long lifetime of positrons, most of the astrophysical (37) as we have mentioned in the introductory chapter. The dust ion-acoustic waves are longitudinal oscillations of the ions (and the electron-positron) in a dusty e-p-i plasma.

To the best of our knowledge, no theoretical investigation has been developed to study electrostatic shock waves under the extreme condition of matter for both non-relativistic and ultra-relativistic limits in a degenerate plasma system with dusty e-p-i plasma. Therefore, in our present investigation, we consider a degenerate dense plasma system in absence of the magnetic field containing non-relativistic degenerate cold ion fluid, both non-relativistic and ultra-relativistic degenerate electrons and positrons where the ion is the heavier element among all other elements. To study the basic features of the dust ion-acoustic nonlinear structures of shock wave in such unmagnetized three component degenerate dense plasma, we have studied the Burgers' equation and the numerical solution of Burgers' equation. The model is relevant to compact interstellar objects (e. g. white dwarf, neutron star, etc.).

2 Governing Equations

We consider an unmagnetized collisionless three component degenerate dense plasma system consisting of non-relativistic cold degenerate ion fluid and both non-relativistic and ultra-relativistic degenerate electron and positron fluids. The dynamics of the one dimensional dust ion-acoustic waves in such a three component degenerate dense plasma system is governed by

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}(n_s u_s) = 0, \tag{2.1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{K_1}{n_i}\frac{\partial n_i^{\alpha}}{\partial x} - \eta \frac{\partial^2 u_i}{\partial x^2} = 0, \tag{2.2}$$

$$n_e \frac{\partial \phi}{\partial x} - K_2 \frac{\partial n_e^{\gamma}}{\partial x} = 0, \tag{2.3}$$

$$n_p \frac{\partial \phi}{\partial x} - K_2 \frac{\partial n_p^{\gamma}}{\partial x} = 0, \tag{2.4}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e \alpha_e - n_i - \alpha_p n_p + \mu, \tag{2.5}$$

where n_s is the plasma number density of the species s $\{s= e$ (for electron), p (for positron), and i (for ion)$\}$ normalized by its equilibrium value n_{so} (n_{e0}), u_s is the plasma species fluid speed normalized by $C_{im} = (m_e c^2/m_i)^{1/2}$ with m_e (m_i) being the electron or positron (ion) rest mass and c being the speed of light in vacuum, ϕ is the electrostatic wave potential normalized by $m_e c^2/e$ with e being the magnitude of the charge of an electron, the time variable (t) is normalized by $\omega_{pi} = (4\pi n_{i0e}^2/m_i)^{1/2}$, and the space variable (x) is normalized by $\lambda_m = (m_e c^2/4\pi n_{s0}^2)^{1/2}$. The coefficient of viscosity η is a normalized quantity given by $\omega_i \lambda_{mi}^2 m_s n_{s0}$, and α_e is the ratio of the equilibrium number density of electron to ion (n_{e0}/n_{i0}), $\alpha_p$is the ratio of the equilibrium number density of positron to ion (n_{p0}/n_{i0}), and μ is the ratio of the equilibrium number density of negatively charged dust grains to ion $(Z_d n_{d0}/n_{i0})$. The constants $K_1 = n_0^{\alpha-1} K_i/{m_i}^2 {C_i}^2$ and $K_2 = n_0^{\gamma-1} K_e/m_i {C_i}^2$=$n_0^{\gamma-1} K_p/m_i {C_i}^2$. The equations of state used here for the degenerate pressures of electrons, positrons, and ions are given by

$$P_i = K_i n_i^{\alpha}, \tag{2.6}$$

where

$$\alpha = \frac{5}{3}; \;\; K_i = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{3}} \frac{\pi \hbar^2}{m} \simeq \frac{3}{5}\Lambda_c \hbar c, \tag{2.7}$$

for the non-relativistic limit (where $\Lambda_c = \pi\hbar/mc = 1.2 \times 10^{-10}$ cm, and $\hbar$ is the Planck constant divided by 2π). While for the electron fluid,

$$P_e = K_e n_e^{\gamma}, \tag{2.8}$$

and while for the positron fluid

$$P_p = K_p n_p^{\gamma}, \tag{2.9}$$

where for non-relativistic limit (26; 27; 28; 32; 33)

$$\gamma = \alpha; K_e = K_p \tag{2.10}$$

and for the ultra-relativistic limit (26; 27; 28; 32; 33)

$$\gamma = \frac{4}{3}; \quad K_e = K_p = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{\frac{1}{3}} \hbar c \simeq \frac{3}{4}\hbar c, \tag{2.11}$$

3 Derivation of Burgers' equation

Now, we derive a dynamical equation for the nonlinear propagation of the dust ion-acoustic shock waves by using (2.1 - 2.5). To do so, we employ a reductive perturbation technique to examine electrostatic perturbations propagating in the relativistic degenerate dense dusty plasma due to the effect of dissipation, we first introduce the stretched coordinates (38)

$$\zeta = \epsilon(x - V_p t), \tag{3.1}$$
$$\tau = \epsilon^2 t, \tag{3.2}$$

where V_p is the wave phase speed (ω/k with ω being angular frequency and k being the wave number of the perturbation mode), and ϵ is a smallness parameter measuring the weakness of the dispersion ($0 < \epsilon < 1$). We then expand n_i, n_e, n_p, u_i, and ϕ, in power series of ϵ:

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots, \tag{3.3}$$
$$n_e = 1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \cdots, \tag{3.4}$$
$$n_p = 1 + \epsilon n_p^{(1)} + \epsilon^2 n_p^{(2)} + \cdots, \tag{3.5}$$
$$u_i = \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \cdots, \tag{3.6}$$
$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \tag{3.7}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, using equations (3.1)-(3.7) into equations (2.1) - (2.5) we get, $u_i^{(1)} = V_p\phi^{(1)}/(V_p^2 - K_1')$, $n_i^{(1)} = \phi^{(1)}/(V_p^2 - K_1')$, $n_e^{(1)} = n_p^{(1)} = \phi^{(1)}/K_2'$, and $V_p = \sqrt{(\frac{K_2'}{\alpha_e - \alpha_p} + K_1')}$, where $K_1' = \alpha K_1/(\alpha - 1)$ and $K_2' = \gamma K_2/(\gamma - 1)$. The relation $V_p = \sqrt{(\frac{K_2'}{\alpha_e - \alpha_p} + K_1')}$ represents the dispersion relation for the dust ion acoustic type electrostatic waves in the degenerate plasma under consideration.

We are interested in studying the nonlinear propagation of these dissipative dust ion-acoustic type electrostatic waves in a four component degenerate plasma. To the next higher order in ϵ, we obtain a set of equations

$$\frac{\partial n_s^{(1)}}{\partial \tau} - V_p \frac{\partial n_s^{(2)}}{\partial \zeta} - \frac{\partial}{\partial \zeta}[u_s^{(2)} + n_s^{(1)} u_s^{(1)}] = 0, \tag{3.8}$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V_p \frac{\partial u_i^{(2)}}{\partial \zeta} + u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \zeta} + \frac{\partial \phi^{(2)}}{\partial \zeta} - \eta \frac{\partial^2}{\partial \zeta^2} u_p^{(1)}$$
$$+ K_1' \frac{\partial}{\partial \zeta}\left[n_i^{(2)} + \frac{(\alpha - 2)}{2}(n_i^{(1)})^2\right] = 0, \tag{3.9}$$

$$\frac{\partial \phi^{(2)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta}\left[n_e^{(2)} + \frac{(\gamma - 2)}{2}(n_e^{(1)})^2\right] = 0, \tag{3.10}$$

$$\frac{\partial \phi^{(2)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta}\left[n_p^{(2)} + \frac{(\gamma - 2)}{2}(n_p^{(1)})^2\right] = 0, \tag{3.11}$$

$$0 = \alpha_e n_e^{(2)} - n_i^{(2)} - \alpha_p n_p^{(2)}. \tag{3.12}$$

Now, combining (3.8-3.12) we deduce a Burgers' equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} = C \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2}, \tag{3.13}$$

where the value of A and C are given by

$$A = \frac{(V_p^2 - K_1')^2}{2V_p}\left[\frac{3V_p^2 + K_1'(\alpha - 2)}{(V_p^2 - K_1')^3}\right.$$
$$\left. + \frac{(\gamma - 2)(\alpha_e - \alpha_p)\beta^2}{{K_2'}^2}\right], \tag{3.14}$$

$$C = \frac{\eta}{2}. \tag{3.15}$$

The shock wave solution of equation (3.13) is

$$\phi^{(1)} = \phi_m[1 - \tanh(\frac{\xi}{\delta})], \tag{3.16}$$

where the special stretched coordinates, $\xi = \zeta - u_0\tau$, the amplitude, $\phi_m = u_0/A$, the width, $\delta = 2C/u_0$ (39; 40), u_0 is the wave speed and the parameter η was chosen from standard value (5) for the system under consideration.

4 Numerical Analysis

We have numerically solved the Burgers' equation (3.13), and the solution of equation (3.13) is equation (3.16). Using the solution, we have studied the effects of μ, η, α_e, and α_p with ξ on dust ion acoustic nonlinear structures (particularly, shock waves) in both non-relativistic and ultra-relativistic degenerate electrons and positrons where ions always being non-relativistic degenerate. It is obvious from figures 1 - 16 that the degenerate plasma system under consideration supports compressive and rarefactive dust ion acoustic shock waves which are associated with both positive and negative potential. The amplitude of these shock waves depends on the non-relativistic and ultra-relativistic limits of the constituent particles.

In figures 1 - 8 we have observed the effects of μ, η, α_e, and α_p on the potential structure with the variation of ξ in case of both non-relativistic and ultra-relativistic limits. It has been observed when the values of α_e is greater than 0.62 ($\alpha_e > 0.62$) then we observe the formation of positive potential for the nonlinear structures of compressive shock wave in our considered degenerate plasma system. Again

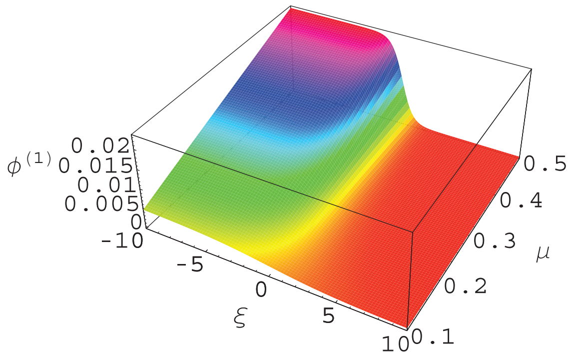

Figure 1: The effect of μ on shock wave for positive potential when e-i-p being non-relativistic degenerate.

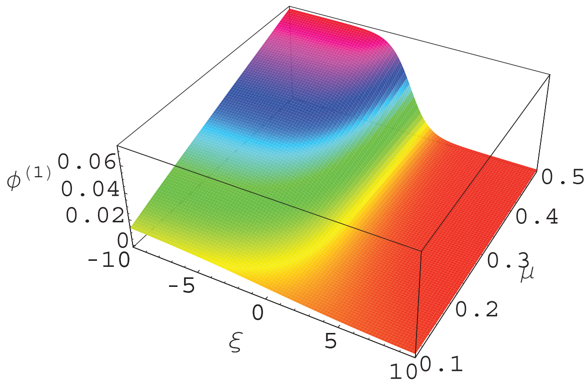

Figure 2: The effect of μ on shock wave for positive potential when $e - p$ being ultra-relativistic and i being non-relativistic degenerate.

we have considered the values of α_e is less than or equal to 0.62 ($\alpha_e \leq 0.62$) then we observe the negative potential for the nonlinear rarefactive structures as shown in figures from 9 to 16. It should be noted here that in both cases we keep all the parameters same so that it could be easy to analysis the effects of every parameter.

To observe the nature of the positive potential in our considered degenerate plasma system, we have studied the effects of the increasing value of μ, η, α_e, and α_p on the potential, $\phi^{(1)}$ with the variation of ξ in both limits from figures 1 to 8. The interesting point is that with the increasing values of μ, α_p, and α_e, the values of potential increases smoothly, but with the increasing values

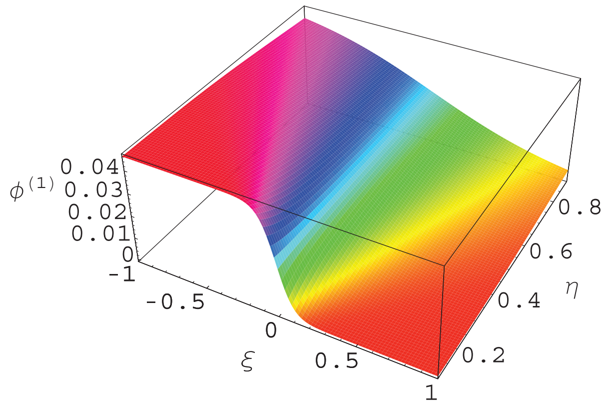

Figure 3: The effect of η on shock wave for positive potential when e-i-p being non-relativistic degenerate.

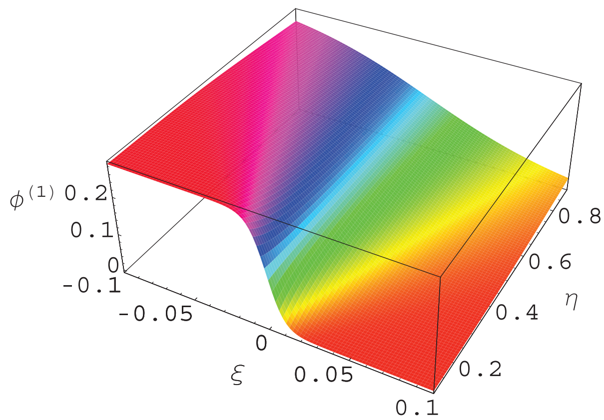

Figure 4: The effect of η on shock wave for positive potential when $e-p$ being ultra-relativistic and i being non-relativistic degenerate.

of η the potential decreases smoothly. It should be noted that the formation of the compressive dust ion acoustic shock waves depends only on the values of the ratio of the electron number density and the ion number density (α_e) and does not depend on the values of μ and α_p (the ratio of the electron number density and the positron number density). When we consider the effect of the potential $\phi^{(1)}$ with ξ, we observe that with the increasing value of ξ ($\xi < 0$) the decrease of potential is not so high. But when the value of ξ is nearly zero, the potential decreases suddenly as the name of the wave exists (i.e. shock). And the figures 1 - 8 also show us that the potential for electron-positron (e-p) being ultra-relativistic and ion (i) being non-relativistic degenerate is always greater than for

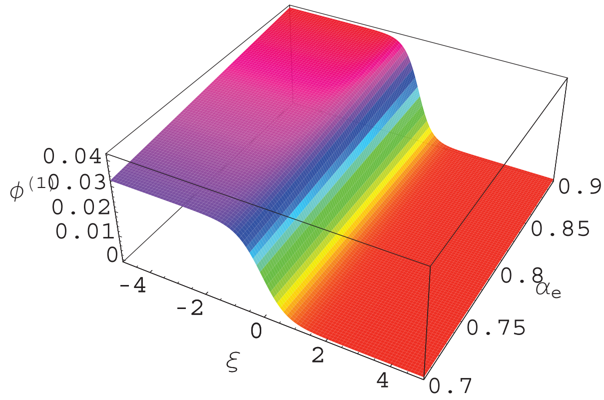

Figure 5: The effect of α_e on shock wave for positive potential when e-i-p being non-relativistic degenerate.

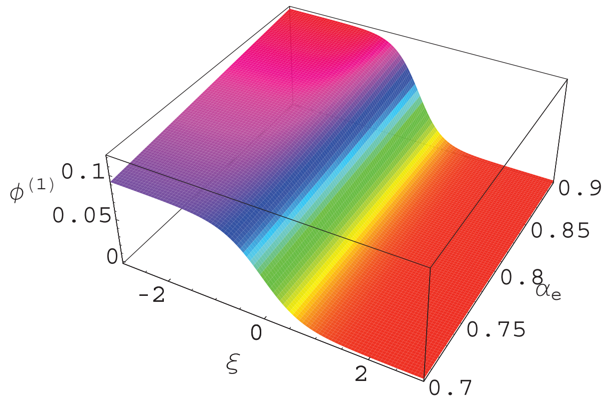

Figure 6: The effect of α_e on shock wave for positive potential when $e - p$ being ultra-relativistic and i being non-relativistic degenerate.

electron-ion-positron (e-i-p) being non-relativistic degenerate.

From the study of the negative potential in our four component degenerate plasma system, we have observed that with the increasing value of μ and η the potential also increases. But it has been also noted that with the increasing values of α_e and α_p, the potential, ($\phi^{(1)}$) also decreases very smoothly in both limits from figures 9 to 16. It should be noted here that in this case, the values of α_e is always less than or equal to 0.62 ($\alpha_e \leq 0.62$). It needs to be pointed here that the formation of the rarefactive dust ion-acoustic shock waves (shown in figures 9 to 16) depends only on the values of the ratio of the electron number density and the ion number density (α_e) and does not depend on the

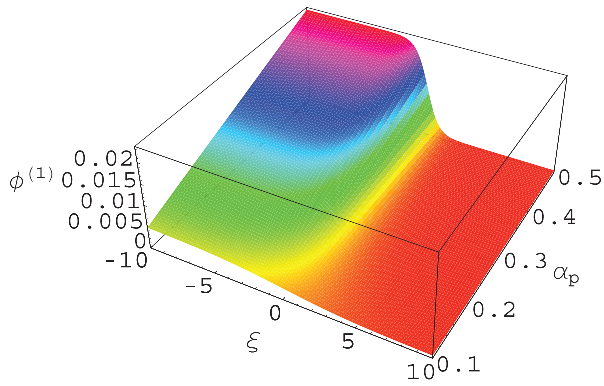

Figure 7: The effect of α_p on shock wave for positive potential when e-i-p being non-relativistic degenerate.

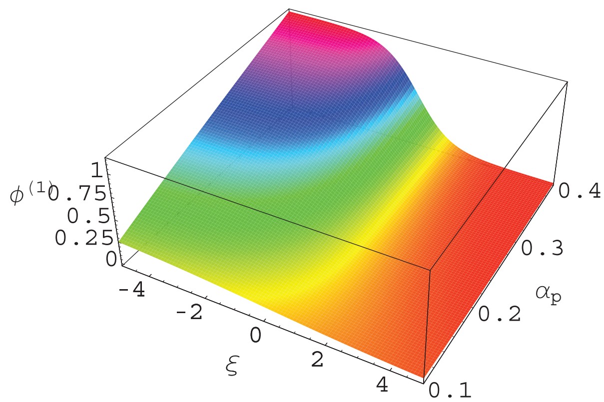

Figure 8: The effect of α_p on shock wave for positive potential when $e-p$ being ultra-relativistic and i being non-relativistic degenerate.

values of μ and α_p (the ratio of the electron number density and the positron number density). From the analysis of the negative potential structures for $\alpha_e \leq 0.62$ (shown in figures 9 - 16) it has been again pointed out that the potential for electron-positron (e-p) being ultra-relativistic and ion (i) being non-relativistic degenerate is always greater than for electron-ion-positron (e-i-p) being non-relativistic degenerate.

It is to be noted here that we have taken all the parameters in normalized form, so all the ranges of parameters are taken arbitrarily.

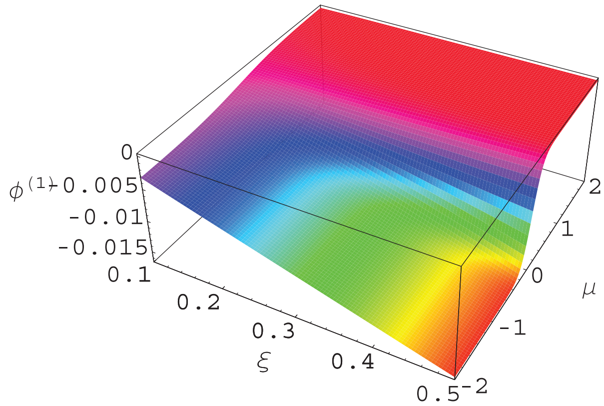

Figure 9: The effect of μ on shock wave for negative potential when e-i-p being non-relativistic degenerate.

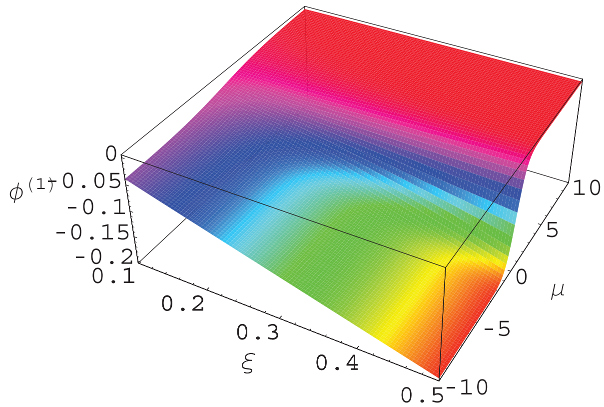

Figure 10: The effect of μ on shock wave for negative potential when $e-p$ being ultra-relativistic and i being non-relativistic degenerate.

5 Conclusion

The reductive perturbation method for Burgers' equation was used to investigate the propagation of DIA shock waves in electron-positron-ion degenerate dense plasma. It was shown that the relativistic degenerate pressure of electrons, ions, and positrons in such plasma has significant effects on the propagation of DIA shock waves in degenerate plasmas. The negatively charged static dust particles participate only in maintaining the quasi-neutrality condition at equilibrium. Calculations reveal that DIA shock waves behave much different in such plasmas under non-relativistic and ultra-relativistic

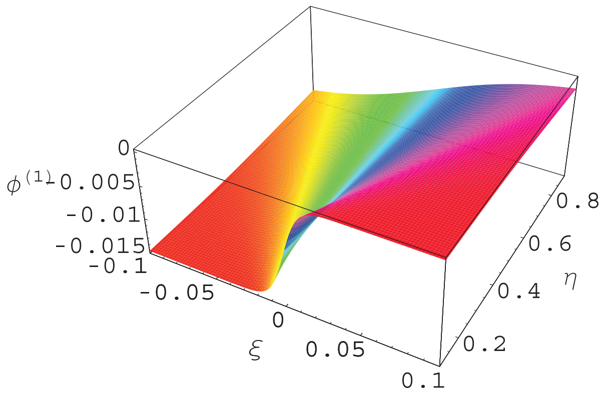

Figure 11: The effect of η on shock wave for negative potential when e-i-p being non-relativistic degenerate.

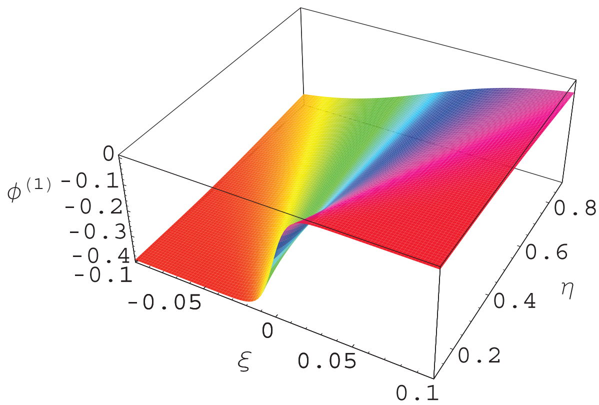

Figure 12: The effect of η on shock wave for negative potential when $e-p$ being ultra-relativistic and i being non-relativistic degenerate.

degenerate pressure. We note that in our numerical analysis we have used a wide range of the degenerate plasma parameters, which are relevant for many cosmic environments and compact astrophysical objects, viz. solar atmosphere, active-galactic-nuclei, pulsar magnetospheres cores of giant planets like Jupiter, primordial universe, neutron stars, white dwarfs, supernovas, etc. (37). The results of the present investigation is, therefore, expected to be useful in understanding the dissipation properties of the electrostatic waves in such cosmic environments, and compact astrophysical objects (white dwarfs, neutron stars etc.) (37). It may be added here that the nonplanar propagation of arbitrary amplitude electrostatic waves in a degenerate electron-ion-positron plasma is also a problem

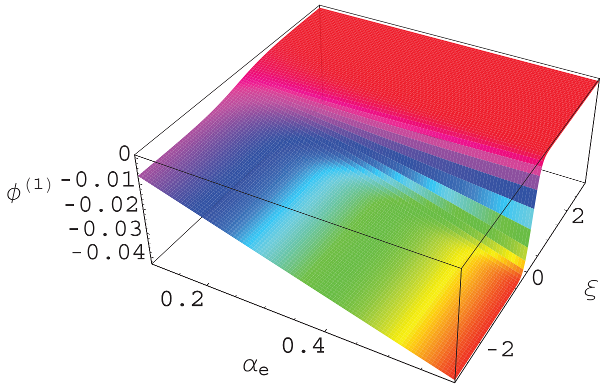

Figure 13: The effect of α_e on shock wave for negative potential when e-i-p being non-relativistic degenerate.

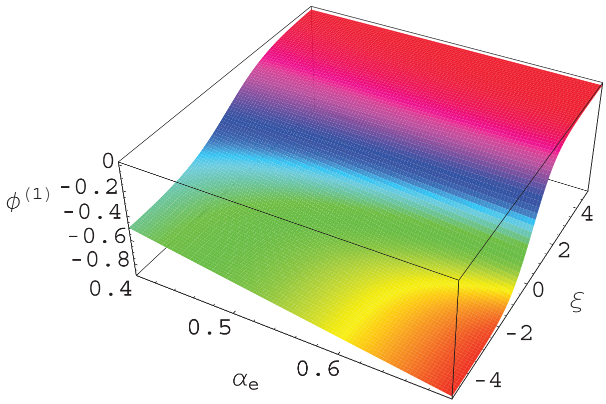

Figure 14: The effect of α_e on shock wave for negative potential when $e-p$ being ultra-relativistic and i being non-relativistic degenerate.

of great importance, but beyond the scope of our present work.

We have considered an unmagnetized degenerate dense plasma containing non-relativistic degenerate cold ions fluid and both non-relativistic and ultra-relativistic degenerate electrons and positrons fluid, and have examined the basic features of the electrostatic nonlinear structures that are found to exist in such degenerate dense plasma. In our present investigation, all the degenerate constituents of the considered dense unmagnetized plasma system follow relativistic limits of pressure. The nonlinear DIA shock waves and its propagation have been described thoroughly with Burgers' equation (3.13) and it's solution (3.16). The effects of different plasma parameters on the nonlinear propagation

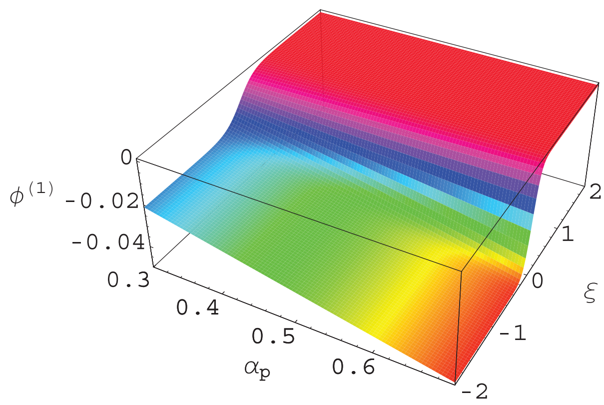

Figure 15: The effect of α_p on shock wave for negative potential when e-i-p being non-relativistic degenerate.

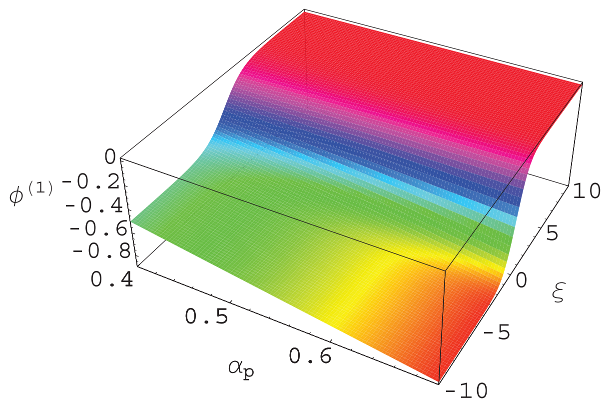

Figure 16: The effect of α_p on shock wave for negative potential when $e - p$ being ultra-relativistic and i being non-relativistic degenerate.

of DIA shock waves have been graphically shown (figures 1 - 16). The degenerate dense plasma is found to support shock structures whose basic features depend on the plasma number density. From this point of view, our present investigation is more acceptable and the system constituents have made the validity of our investigations unique. The electrostatic waves in an ultra-relativistic and non-relativistic degenerate dense plasma, which is relevant to interstellar compact objects like white dwarfs, have been investigated. The results, which have been found from this investigation, represent dust ion acoustic type of electrostatic waves in which the restoring force comes from the electron-positron-ion degenerate pressure and inertia is provided by the ion mass density.

It can be expected that the basic features and the underlying physics of DIA shock waves with the existence conditions for positive and negative potential, that have been presented in our present work should be verified by further laboratory experiments. It may, therefore, be proposed to perform a laboratory experiment which will be able to identify the special new features of DIA shock waves in a four component degenerate dense plasma system that have been predicted in this investigation. It may also be added here that our investigation is valid for small amplitude DIA shock waves and for unmagnetized and uniform degenerate dense plasma system. These results may be useful to explain some aspects of shock waves in dense e-p-i degenerate plasma. However, arbitrary amplitude DIA shock waves in uniform/nonuniform three component degenerate plasma with or without the effects of dust and external magnetic field are also problems of recent interest for many space and laboratory dusty plasma situations, but beyond the scope of our present investigation. Although such plasma cannot be produced in laboratory yet they are gaining considerable attention of the researchers working on dense astrophysical plasma and numerical simulations. We hope that our present investigation will be helpful for understanding the basic features of the localized electrostatic disturbances in compact astrophysical objects (e.g. white dwarfs, neutron stars, etc.).

Competing interests

The authors declare that no competing interests exist.

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Appendix for the solution of Burgers' Equation

The Burgers' Eq. can be written as

$$\frac{\partial y}{\partial t} + Ay\frac{\partial y}{\partial x} = C\frac{\partial^2 y}{\partial x^2}, \tag{5.1}$$

where A and C are constants. To obtain a stationary localized solution of this Burgers' Eq., we first transform the independent variables to $\xi = x - U_0 t$, $\tau = t$:

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} - U_0\frac{\partial}{\partial \xi}, \tag{5.2}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}. \tag{5.3}$$

Now, substituting Eqs. (5.2) and (5.3) into Eq. (5.1), one gets

$$\frac{\partial y}{\partial \tau} - U_0\frac{\partial y}{\partial \xi} + Ay\frac{\partial y}{\partial \xi} = C\frac{\partial^2 y}{\partial \xi^2}. \tag{5.4}$$

For steady state condition ($\partial y/\partial \tau \to 0$) Eq. (5.4) reduces to

$$-U_0\frac{dy}{d\xi} + Ay\frac{dy}{d\xi} = C\frac{d^2 y}{d\xi^2},$$

$$\Rightarrow \quad -U_0 y + \frac{1}{2}Ay^2 - C\frac{dy}{d\xi} = C_1, \tag{5.5}$$

where C_1 is an integration constant. Now, under appropriate boundary conditions, viz. $y \to 0$ and $dy/d\xi \to 0$ at $\xi \to \infty$, one can find C_1 as $C_1 = 0$. So, the above Eq. becomes

$$\frac{dy}{d\xi} = \frac{1}{C}\left(-U_0 y + \frac{1}{2}Ay^2\right). \tag{5.6}$$

$$\frac{dy}{d\xi} = -\frac{Ay}{2C}\left(\frac{2U_0}{A} - y\right). \tag{5.7}$$

We now let

$$\alpha = \frac{A}{2C}, \tag{5.8}$$

$$\beta = \frac{2U_0}{A}, \tag{5.9}$$

$$now, \quad \gamma = \alpha\beta,$$

$$\Rightarrow \quad \gamma = \frac{A}{2C}.\frac{2U_0}{A},$$

$$\Rightarrow \quad \gamma = \frac{U_0}{C}. \tag{5.10}$$

Now, using Eqs. (5.8), (5.9), and (5.10) into Eq. (5.7), one obtains

$$\frac{dy}{y(\beta - y)} = -\alpha d\xi. \tag{5.11}$$

Integrating both sides of Eq. (5.11), we get

$$\Rightarrow \quad \ln\left[\frac{y}{\beta - y}\right] = -\gamma\xi + C_2,$$

$$\Rightarrow \quad \frac{y}{\beta - y} = C_3 e^{-\gamma\xi},$$

$$\Rightarrow \qquad y = \frac{C_3 \beta e^{-\gamma\xi}}{1 + C_3 e^{-\gamma\xi}}. \tag{5.12}$$

where C_2 is an integration constant and $C_3 = e^{C_2}$. Now, imposing the condition, $y = \beta/2$ at $\xi = 0$, one can find

$$\frac{\beta}{2} = \frac{C_3 \beta}{1 + C_3},$$
$$\Rightarrow \quad C_3 = 1.$$

Using $C_3 = 1$ in Eq. (5.12), we get

$$\Rightarrow \quad y = \frac{\beta e^{-\gamma\xi}}{1 + e^{-\gamma\xi}},$$
$$\Rightarrow \quad y = \frac{\beta}{2} . \frac{2e^{\frac{-\gamma\xi}{2}}}{e^{\frac{\gamma\xi}{2}} + e^{\frac{-\gamma\xi}{2}}},$$
$$\Rightarrow \quad y = \frac{\beta}{2} \left[1 - \frac{e^{\frac{\gamma\xi}{2}} - e^{\frac{-\gamma\xi}{2}}}{e^{\frac{\gamma\xi}{2}} + e^{\frac{-\gamma\xi}{2}}} \right]. \tag{5.13}$$

Therefore, the stationary shock wave solution of the Burgers' equation is

$$y = y_0 \left[1 - \tanh \frac{(x - U_0 \tau)}{\Delta} \right], \tag{5.14}$$

where y_0 and Δ are the amplitude and the width of the shock waves, respectively, and are given by

$$y_0 = \frac{U_0}{A}, \tag{5.15}$$
$$\Delta = \frac{2C}{U_0}. \tag{5.16}$$

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