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The Origin of Quantification

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Neither the Greek nor the Alexandrian nor the early Arabic philosopher/scientists ever developed a mathematical representation of qualities, a prerequisite for a mathematical physics. By the early seventeenth century the quantification of qualities was a common practice. This article traces the way this practice developed. It originated with a medievally theological problem and was developed by philosophical logicians who did not have mathematical physics as a goal. The verbal algebra they developed was given a mathematical formulation in the late fifteenth century. This was subsequently assimilated into a neo-Platonic revival that stressed mathematical forms. The quantification of qualities developed in physics supplied the paradigm for quantification in other fields.

Keywords: Quantification; Early History of Physics

Introduction

Greek and Alexandrian scientists developed geometry and arithmetic. Yet, neither they nor the Arab successors ever developed a quantitative science of qualities. At the start of the scientific revolution in the early 17th century, a quantitative treatment of qualities was a common practice. We will consider what prepared the way for this practice (MacKinnon, 2011: chap. 2). It helps to begin by considering why the Greeks and Alexandrians did not have this practice.

Historical Development

Two basic reasons might explain this deficiency. The first stems from the Greek understanding of the relation of mathematics to reality. For Plato mathematical forms exist in a separate realm of ideas. For Aristotle mathematics is derived from physical reality by a process of concept abstraction. Since his position supplies the background for the medieval philosophers we will consider, it requires a more detailed treatment.

In his *de Interpretation* (chap. 1) Aristotle declared that though different languages have different word to symbolize mental experiences, all men have the same mental experiences. The alleged reason is that the things determine our concepts of them. Aristotle's ordered list of categories: substance, quantity, quality, relation, place, time, situation, state, action, and passion, provide the most basic concepts. The first three categories have a conceptual ordering that determines the way quantities are treated. Consider the predicate "red". This cannot be applied in a literal sense to an immaterial being, such as an idea, or an unextended being, such as a point. A quality, such as color or taste, pre-supposes extension which, in turn, presupposes a substance that is extended and colored. Any discussion of the quantity of a quality perverts the proper conceptual ordering.

In addition to speculative difficulties, there were practical difficulties. The Greeks and Romans represented integers by arbitrarily chosen letters. This blocked any mathematical representation of continuously varying qualities. It also made it extremely difficult to develop any sort of isomorphism between numbers as a system and the quantity of qualities. Numbers were generally treated in a way that obscured the logical structures needed. In the Pythagorean tradition numbers were classified as even and odd, and then into evenly even (powers of two), evenly odd, and oddly even, with a further distinction into prime, composite and perfect numbers. This supported the popular trend seeking properties of particular numbers that were associated with particular qualities, rather than the properties of numbers as a system.

Aristotle's doctrine of categories were well known to medieval scholars even before the systematic translation of Aristotle's works in the late twelfth and early thirteenth century. Porphyry, a disciple of Plotinus, wrote a commentary on the *Categories*, the *Isagoge*. This was translated into Latin, Syrian, Arabic, and Armenian and served as a staple text for the early arguments between Nominalists and Realists. The basic categories were accepted as something determined by the nature of reality. This put analysis in ontological terms. Since the basic categories as well as concepts of particular objects are determined by the reality known, one can analyze objects by analyzing concepts of objects. This linguistic analysis was carried on in a theological perspective.

This theological perspective seemed to require a quantification of qualities. One's rank in heaven, according to accepted teaching, depends upon the degree of grace, or charity, that one has at the moment of death. Dante's *Divine Comedy* vividly illustrates the different places assigned in hell, purgatory, and heaven. Since grace is a quality, albeit a supernatural one, comparing degrees of grace is comparing quantities of qualities. Accordingly, a way had to be found to discuss the quantity of a quality. St. Thomas Aquinas, the most Aristotelian of the medieval philosophers, seems to have been the first to give a coherent account of the way in which quantitative determinations can be given to qualities. Aquinas distinguished between quantity *per se*, or bulk quantity, and quantity *per accidens*, or virtual quantity. (Summa Theologiae, I, q. 42, a.1, ad 1) Virtual quantity, or the quantity of a quality, can have magnitude by reason of the subject in which it inheres, as a bigger wall has more whiteness than a smaller one, or it can have magnitude by reason of the effect of its form. The first effect of a form is a way of existing, e.g., as human. The secondary effect of a form is shown through its action on objects. A comparison of relative effects serves as a measure of virtual quantity. Thus, one with greater strength can lift heavier rocks.

Measurement did not rest on the modern idea of a unit of measure or even a practice of measuring things. In medieval thought the Platonic idea that the perfect form is the measure for any being that participates in that form was reinforced by the scriptural statement that in creating the world God disposed all things in number, weight, and measure. Even when measurement is separated from a doctrine of participation and treated in terms of numbers, as in Aristotle's treatment of time, the constraint is that everything must be measured "by some one thing homogeneous with it, units by a unit, horses by a horse, and similarly times by some definite time" (Physics, 223b14). Applying quantities to qualities broke this Aristotelian constraint. This new idea of the quantity of a quality was the pivot leading from the Aristotelian philosophy of nature to a mathematical physics.

The historical development of medieval natural philosophy has been treated in detail elsewhere (Lindberg, 1992: chap. 12; Clagett, 1959). In summaries, this is often done by presenting the aspects leading to Newtonian physics in a manner intelligible to a modern audience. Here I wish to do the opposite, to bring out the complexities and confusion involved in developing an account of properties of matter and motion that admitted of a mathematical representation. The evolution of the quantification of qualities followed the general evolutionary pattern of advancing by coping with particular problems and without any goal of developing a mathematical physics.

The idea of the quantity of a quality matured into a doctrine of the intensification and remission of qualities. This led to a nest of conceptual problems. Does the quality itself change, the degree of participation in a quality, or does one quality replace another? If the quality changes by addition, rather than replacement, how is the addition of qualities to be understood?

The nominalism, spearheaded by William of Ockham, led to a de-emphasis on the ontological aspects of this discussion. Instead of asking how intensification and remission of a quality takes place he sought a criterion allowing one to predicate "strong" or "weak" of the qualities a thing has.

The mathematization of this came chiefly from the "Calculators" of the Merton school in fourteenth century Oxford and later from the Parisian school. What mathematics did they have (Mahoney, 1987)? In the twelfth century, three new elements were introduced and gradually assimilated: Hindu-Arabic arithmetic with its superior notation, Euclidean geometry, and the algebra in the first (of three) parts of al-Khwarizmi's treatise. This part ended with the rule of three, or how to infer a fourth number on the basis of three. Thus, if eight cost five, how much do eleven cost? It was a verbal algebra. No symbols were used even for numbers. Jordanus Nemorarius in the thirteenth century first introduced these, in a very limited way. This treatment of proportions was gradually fused with Euclidean geometry. Euclid, more an organizer than an originator, had two distinct theories of ratios and proportions. The one in Book VII, stemming from Pythagoras, was limited to integers. The one in Book V, stemming from Eudoxos, treated continuous magnitudes. The mediaevals who adapted this lacked Eudoxos's concern with incommensurables and existence theorems. From these elements, they fashioned conceptual tools that could treat intensification of quantities through a kind of verbal algebra.

To grasp the conceptual problems it is important to change perspectives. Today we would express the rule of three in a ratio which we would symbolize as A/B = C/D. The Merton calculators would symbolize this rule as (A,B) = (C,D). What is significant is not the change in format, but the interpretation given to it. The terms we treat as numerators and denominators were understood as parts of a system of classification which admitted of groups and sub-groups. If A is a multiple of B then (A,B) is a *multiple ratio*. If A contains B once with a remainder of 1 then (A,B) is a *superparticular* ratio. This admits of different kinds. (3,2) is a *sesquialterate* ratio; (4,3) is a *sesquitertian* ratio. If A contains B once with a remainder greater than one, then (A,B) is a *superpartient* ratio, which also admits of sub-groups. If A contains B more than once, then one has the general categories of *multiple superparticular* ratios and *multiple superpartient* ratios. In this way, Euclid's system of proportions gradually became assimilated to the arithmetic of fractions. This was extended to ratios of ratios, but still using terms and categories rather than mathematical symbols for numbers. Thus a limited verbal algebra was developed for expressing proportions for quantities of qualities. It could not yet treat a continuous variation in the quantity of a quality.

This systematization was given a kind of state-space representation in the late fifteenth century by Nicole Oresme at the University of Paris. Since this represents the transition from a verbal algebraic treatment of quantification to the beginning of a mathematical treatment is deserves more consideration. In interpreting this it is important to realize the very abstract level at which the physical-mathematical correspondence is found. I will present Oresme's explanation of this and then comment on it.

Every measurable thing except numbers is to be imagined in the manner of continuous quantity. Therefore for the mensuration of such a thing, it is necessary that points, lines, and surfaces, or their properties be imagined. For in them (i.e., the geometrical entities), as the philosopher has it, measure or ratio is initially found, while in other things it is recognized by similarity as they are being referred by the intellect to them (i.e., to geometrical entities). Although indivisible points, or lines, are non-existent, still it is necessary to feign them mathematically for the measures of things and for the understanding of their ratios. Therefore, every intensity which can be acquired successively ought to be imagined by a straight line perpendicularly erected on some point of the space or subject of the intensible thing, e.g., a quality. For whatever ratio is found to exist between similar ratio is found to exist between line and line, and vice versa (Clagett, 1966: pp. 165-167).

Consider a body and a variable quality, such as motion or heat. Represent the quality by a base line (eventually an x-coordinate) and the intensity by a perpendicular (or y-coordinate). "Measurement", as Oresme uses this term, does not presuppose any unit or method of measurement. The length of the lines representing intensities has no absolute significance. What counts are the ratios. If the intensity doubles then the length of the perpendicular line should double. The initial assignments are always arbitrary, e.g. "Let the intensity of a quality have a value of four". As the intensity of a quality changes, the ratio of line lengths representing the ratio of the changing intensities also change. The mathematics of proportions handled ratios.

The men who developed these systems were, by our occupational categories, logicians, not physicists. They developed logical systems that admitted of mathematical representation and which might, incidentally, admit of physical examples. The term "motion" still meant change in a quality with local motion gradually emerging as the most significant type and "velocity" a term for the intensity of local motion. Wallace (1981) has clarified the basic systematizations used. I will use his notation. A uniform motion (U) is one with a constant velocity (v), while a difform motion (D) has changing velocity. Motion may be uniform or difform in two different ways: with respect to space (U(x) or D(x)) depending on whether all the parts of a body move with the same velocity; and with respect to time, U(t) or D(t). Difform motion is of various kinds. Motion that is difform with respect to the parts of the object moved may be uniformly difform, U(x), in the sense that there is a uniform spatial variation in the velocity of the various parts of the body, or difformly difform, D(x), if there is no such uniformity, or it may be uniformly difform, U(t), or difformly difform, D(t), with respect to time. This allows of various schema for treating velocity. I will outline the one that Oresme used (**Figure 1**).

The standard example of (3) is a rotating wheel, or, for Oresme, the heavens, as circling around the earth or the sun (a hypothesis he considered). The different parts move at different velocities, but do not change in time. An example of (2) is a falling stone. All the parts move at the same velocity, but the overall velocity increases. This is not as good an example, since it is not really uniformly difform. The increase in velocity gradually stops, especially for light bodies. Either (2) or (3) may be represented by a diagram which illustrates the Merton theorem on uniformly difform motion. AC is the time axis, while

BF and CD represent intensities of uniformly difform motion. The quantity of motion is represented by the area under the line representing motion. Flipping the triangle, FGD, over so that D coincides with C produces the rectangle, ACGE. This demonstrates that the total motion of a uniformly difform motion is equal to the total motion of a uniform motion with half the terminal velocity. It also follows that the total motion in the second interval is three times the motion in the first interval. This is the aspect Galileo adapted in treating acceleration.

This abstract approach raised questions that could not be handled in the received Aristotelian terminology. Thus, half the terminal velocity meant the instantaneous velocity at the point,

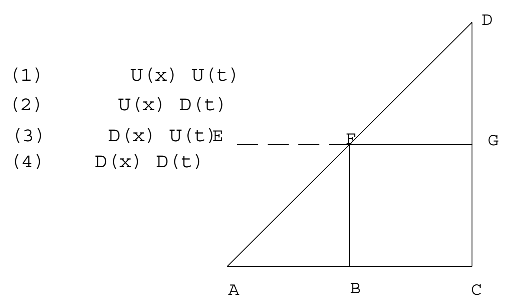

Figure 1.
Oresme's representation of the Merton theorem of uniformly difform motion.

F. This seems to involve vanishingly small distances and times. The development of this path led to Newton's theory of fluxions, or differential calculus. The notion of quantity of motion was still obscure. However, it was represented by the area under a line. For uniformly difform motion the areas involved triangles and rectangles, something these medieval logicians could handle. However, for difformly difform motion the quantity of motion is the area under an irregular curve. Archimedes had treated some such areas by his method of exhaustion. His works were not yet available. The development of this path led to the integral calculus of Leibnitz and his account of functions. For Leibnitz a function expressed the relation of a dependent variable to an independent variable. The ultimate independent variables are space and time. Functions, so defined, express the relations represented in Oresme's diagrams.

In the sixteenth century, Italy became the center for the development of mechanics, and of a mechanics that became more clearly related to dynamics. There were three somewhat separate traditions. The first, an academic tradition, was an extension of the work of the Calculators. Paul of Venice studied at Oxford and then, on his return, taught Mertonian ideas (Wallace, 1981: Part II). Here the treatment of motion was caught up in the traditional clash between Nominalists and Realists. This concern influenced the dynamic tradition that Galileo redeveloped. The Nominalists focused on the new mathematical treatment and slighted the realistic significance of their formalism. The other two traditions, developed apart from the universities, were strongly influenced by the publication, in 1453 and later, of Moerbeke's translation of some of Archimedes' works. The Northern group, Tartaglia, Cardano, and Benedetti, was very concerned with applications of mechanics to ballistics. The group in central Italy, Commandind, Ubaldo, and Baldi, were more mathematically oriented. (Drake & Drabkin, 1969).

After the fall of Constantinople (1453) some Greek scholars moved to Italy and taught Greek literature and philosophy in the Academies sponsored by some noble families. This led to a type of Neo-Platonism opposed to the Aristotelianism taught in the universities. The Neo-Platonists especially studied Plato's Timaeus, where the astronomer, Timaeus, presents an extended explanation (29e - 92c) of the order of the universe, beginning with the Demiurge fashioning preexistent matter in accord with mathematical forms and proportions and terminating with an explanation of illness and disease resulting from a lack of proportion of the four elements. A fusion of this idea of the universe as an embodied mathematical system with the Biblical account of creation supported the idea that God created the world in accord with ideal mathematical forms. The further fusion of these rather nebulous ideas with the mathematical treatment of quantities of qualities led to the conclusion that in coming to know things through their proper mathematical forms human knowledge matched divine knowledge.

In the Preface to his *Mysterium Cosmographicum*, Kepler wrote: "The ideas of quantities have been in the mind of God from eternity, they are God himself; they are therefore also present as archetypes in all minds created in God's likeness" (Cited from Koestler, 1960: p. 65). In a letter to a friend he said: "For what is there in the human mind besides figures and magnitudes. It is only these which we can apprehend in the right way, and if piety allows us to say so, our understanding is in this respect of the same kind as the divine, at least as far as we are able to grasp something of it in our mortal life. Only fools fear that we make man godlike in doing so; for the divine counsels are impenetrable, but not his material creation" (Citation from Baumgardt: p. 50).

Galileo did not share Kepler's mathematical mysticism. Yet, he assumed a similar relation between quantitative forms as archetypes in the mind of God and men: "I say that human wisdom understands some propositions as perfectly and is as absolutely certain thereof, as Nature herself; and such are the pure mathematical sciences, to wit, Geometry and Arithmetic. In these Divine Wisdom knows infinitely more propositions, because it knows them all, but I believe that the knowledge of these few comprehended by human understanding equals the Divine" (Galileo: p. 114).

As these citations indicate, physical explanations were still functioning in a theological context that had been stretched to accommodate the mathematical representation of qualities. This was regarded as objective, representing reality as it exists independent of our knowledge of it. There was, however, the beginning of a shift from a theological perspective to an observer-centered perspective. Early in the 15th century, the Florentine architect and engineer, Filippo Brunelleschi, developed the basic laws of linear perspective, reportedly by painting a copy of part of the cathedral on top of its mirror image. Massaccio, della Francesca, and others transformed painting by making perspective basic. Leon Batitista Alberti codified the rules of perspective in his book, *Della pittura*, (1436) with a vanishing point and a horizon, both determined by the position of the observer. This is a geometrical representation of space. In linear perspective, the two dimensional representation of a three-dimensional space is thought of as a projection on a two dimensional surface of light rays travelling from the source to the eye of the observer, rather than the flat space of medieval painters. The space represented is a Euclidean homogeneous space organized from the standpoint of an outside viewer. In spite of strenuous opposition this new way of organizing representations of reality from the perspective of an outside observer rapidly spread to other fields. (Frey, 1981; Chevalley, 1993).

Classical French drama respects the "Aristotelian" dramatic unities of an integrated story completed in one day at one locale. Aristotle had only insisted on unity of action. The "classical Aristotelian doctrine" was articulated by sixteenth century Italian critics influenced by perspective. The dramatic action should be presented from the perspective of an observer. Perspective spread to physics when Kepler, influenced by Dürer's perspectival methods as well as Galileo's account of his telescope, showed, in his *Dioptrice*, how a correct geometrical analysis of light rays explained vision. The theory it replaced, Aristotle's doctrine of transmitted images received as impressed sensible species, was never able to account for the fact that distant objects look smaller. Descartes' *La Dioptrique* extended Kepler's work by giving a correct law of refraction. He explained different colors in terms of light producing different pressures on the eyeball. In his earliest experiments Newton refuted this by inserting a stick behind his own eyeball and exerting pressure.

Perspective entered mathematics with Descartes' analytic geometry and the representation of bodies through coordinates in Euclidean space. Most analyses of this focus on the fact that the geometry is Euclidean. What was also novel was the portrayal of space from the perspective of an outside observer. The idea of the detached observer regarding physical reality from an external viewpoint culminates in Descartes' *Discourse on Method* and *Meditations*. This detached-observer view of reality was gradually transformed into the notion of classical objectivity that Husserl sharply criticized.

Conclusion

Regardless of whether gunnery practice or perspective was the primary factor relating the observer to the described motion, the final result is clear. In place of the abstract ratios of the Calculators, the new methods begin with a three dimensional space, which supplies a framework for the measurement of motion. The bodily presence of the subject anchors this framework. Galileo extended this through his development and use of the telescope, describing in precise detail the positions of the Medicean stars (Jupiter's moons) as he saw them on January evenings in 1610. Galileo's work is not simply an extension of the preceding developments. Galileo played a pivotal role in developing empirical science, shaping and judging mathematical formulas by the way they fit controlled observations. In spite of his early exposure to Aristotelian natural philosopher, he took Archimedes, the prototypical mathematical scientist, as his ideal. However, these advances were only possible because of the developments we have been surveying. The mathematical treatment of motion and forces emerged from three centuries of muddling through quantities of qualities, verbal algebra based on proportions, and a gradual switch from a theological interpretative perspective to an observer-centered viewpoint. This quantification of qualities gradually spreads to other fields, something treated by other authors in this issue.

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