



Game Theoretic Analysis of School and Family Decision-Making in the Context of China's after-School Services after the Double Reduction Policy

Lijuan Hu ^{a*} and Qiao Qiu ^a

^a School of Mathematics and Finance, Hunan University of Humanities, Science and Technology, Loudi Hunan, 417000, P.R., China.

Authors' contributions

This work was carried out in collaboration between both authors. Author LH designed the study and managed the analyses of the study, author QQ wrote the first draft of the manuscript. Both authors read and approved the final manuscript.

Article Information

DOI: <https://doi.org/10.56557/jogress/2024/v18i48818>

Open Peer Review History:

This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: <https://prh.ikpress.org/review-history/12298>

Original Research Article

Received: 04/06/2024
Accepted: 08/08/2024
Published: 12/08/2024

ABSTRACT

This paper employs game theory to dissect the interactions between schools and families in the provision and uptake of after-school services, particularly in the wake of China's "Double Reduction" policy. Through an in-depth survey conducted in Loudi, Hunan Province, we find that the importance of educational outcomes, cost implications, resource allocation, and parental satisfaction as the determinants of participation in after-school services. The static and dynamic

*Corresponding author: E-mail: 1023236794@qq.com;

game analyses elucidate the decision-making complexities, highlighting the Nash equilibrium where both schools and families opt to participate in services, indicating a mutual recognition of benefits. Our findings suggest that clear communication, strategic policy support, and community involvement are pivotal for optimizing after-school services. Our research offers actionable insights for policymakers and educational stakeholders, advocating for a balanced approach that considers both the immediate and long-term interests of students, families, and schools.

Keywords: After-school service; game theory; school and family; static and dynamic game analyses.

1. INTRODUCTION

With the implementation of the “Double Reduction” policy, after-school services in primary and secondary schools are gradually becoming an important part of the education system. At present, after-school services have been widely promoted across the country, with the aim of reducing students’ homework burden, enriching students’ extracurricular life, alleviating the pressure of parents’ pick-up and drop-off, and providing more equitable educational opportunities. The decision-making process regarding participation in these services is a complex interplay between schools and families, each with their unique set of objectives, constraints, and expectations. In this paper, we endeavor to study this intricate relationship through game theory, providing a comprehensive analysis of the strategic interactions between schools and families.

With the burgeoning demand for quality educational support that extends beyond the traditional school hours, the provision of after-school services has emerged as a critical component of the educational ecosystem. However, the decision to participate in such services is influenced by a myriad of factors including educational outcomes, cost implications, resource allocation, and parental satisfaction. As mentioned in [1], during the implementation of after-school services, various stakeholders such as the government, schools, students, teachers, parents, and off-campus hosting organizations are involved. It is necessary to coordinate the interests of all parties to promote the healthy development of after-school services. From the perspective of Actor-Network Theory, this study constructs an “Actor-Network” analytical framework for the collaborative supply of after-school services and provides an in-depth analysis of the internal operation mechanism of the collaborative supply of school after-school services [2]. From the perspective of game theory, the authors in [3] conducted a theoretical analysis of the game between the government and social forces and

concluded that increasing government financial investment can lead to the maximization of the overall benefits of the cooperative supply between the government and social forces. The author in [4] proposed improvement strategies from the perspective of collaborative governance among the government, families, schools, and society, aiming to promote the diversification of after-school services. The author in [5] analyzed the interests from the perspective of the power-interest matrix, identified policy implementation deviations and four main interest conflicts, and thus proposed strategies for improving after-school services. To elucidate the dynamics at play, this study constructs a game-theoretic model that encapsulates the various strategy combinations available to schools and families, and explores the benefits and outcomes of each.

When constructing a game-theoretic model, it is essential to take into account a variety of factors involved in participating in after-school services to determine the payoff matrix. Several scholars have delved into these factors, such as [6,7]. Through an in-depth survey conducted in Loudi, Hunan Province, China, this research uncovers the current state of after-school services in rural and urban schools. The findings from the survey serve as a springboard for the game-theoretic analysis, offering empirical evidence of the challenges and opportunities in the realm of after-school services. This article aims to reveal the key factors that influence the decision-making of schools and families, and identify the optimal strategies that can enhance the quality and efficiency of after-school services.

By the conclusion of this paper, we anticipate providing theoretical support and practical suggestions for all stakeholders involved in the after-school service system. It is our hope that the insights gleaned from this study will contribute to a more nuanced understanding of the decision-making processes, ultimately leading to an improved framework that serves the best interests of students, families, and educational institutions alike.

The rest of this paper is delineated as follows: Section 2 presents an overview of the current state of after-school services in primary and secondary schools, based on in-depth surveys conducted in Loudi, Hunan Province, China. In Section 3, we employ game theory to analyze the roles and strategic choices of schools and families, construct a payment matrix to quantify benefits and costs, and explore both static and dynamic game analyses to understand decision-making complexities. In the last section, we discuss the implications of our findings, highlighting the necessity for continuous improvement and policy support to enhance after-school services.

2. CURRENT STATUS OF AFTER-SCHOOL SERVICES

In the past two years, we conducted an in-depth survey on after-school services in some primary and secondary schools in Loudi, Hunan Province, China. Through our research, we have obtained some valuable conclusions.

2.1 Polarization Phenomenon in Rural Schools

The survey results show that the current state of after-school services in rural schools exhibits a trend of polarization [8,9]. On one hand, despite a shortage of teaching staff, some schools have organized teachers to learn and expand their second and third skills during their spare time, such as calligraphy, painting, and handicrafts, with each teacher proficiently mastering two skills on average.

The schools have launched up to 50 special interest groups for after-school services, and the quality of these services has been recognized by students, parents, and society, with high participation enthusiasm from both students and teachers. Teachers have seen an increase in income through after-school services, which has also enhanced their sense of happiness. Some schools have also launched special after-school services that align with their unique characteristics, such as football classes and agricultural classes. The football classes have gained some national fame through professional training and competitions. The agricultural classes utilize local agricultural resources, allowing students to participate in planting and harvesting, thus cultivating their work values and sense of responsibility.

On the other hand, due to limitations in teaching staff and resources, the quality and participation in after-school services in some rural schools have been declining year by year. This not only affects the students' learning experience but also reduces the satisfaction of parents and society.

2.2 Constraints and Limitations in Urban Schools

During the research process, a peculiar phenomenon was also observed: in urban primary and secondary schools with an ample supply of teaching staff, due to space limitations, most after-school service activities are conducted within classrooms, making it difficult to reflect diversity. This situation, though seemingly paradoxical, actually reflects the limitations of urban schools in terms of spatial resources. Despite the abundance of teaching resources, the lack of sufficient activity venues and facilities results in a more monotonous content and form of after-school services, which fails to meet the diverse interests and developmental needs of students. This limitation not only restricts the quality and effectiveness of after-school services but also affects the overall evaluation of these services by students and parents.

3. GAME THEORY ANALYSIS

3.1 Roles of Both Parties

From the perspective of game theory, schools and families are the two main participants in after-school services, forming a complex interactive relationship that can be seen as a game situation.

Firstly, schools are the direct providers of after-school services, bearing the dual responsibilities of policy implementation and service quality assurance. On one hand, the after-school services offered by schools comply with the guidelines and policy requirements of the educational authorities. On the other hand, schools need to provide high-quality after-school services, including professional teacher guidance, a rich curriculum, and a conducive learning environment. By offering quality after-school services that meet the expectations and needs of parents, schools can gain positive evaluations and support from parents, thereby enhancing their reputation in the community and the educational sector.

Secondly, families are the recipients of after-school services, with parents acting as decision-makers and children as the direct beneficiaries. Parents must weigh the costs of after-school services against the benefits their children will receive. They need to decide whether to enroll their children in after-school services and, if so, which type of service would be most suitable. Parents also have the role of providing feedback to schools, suggesting improvements or making demands. It is crucial for schools to communicate clearly about the after-school services offered, while families should articulate their needs and expectations.

The decisions made by schools and families are interdependent. The choices of families can influence the supply of services provided by schools, and vice versa. This dynamic creates a complex interplay where both parties must navigate a balance between cooperation and potential conflict. In the pursuit of maximizing their respective interests, schools and families may need to make compromises and collaborate in certain areas. Effective communication and mutual understanding are key to achieving a harmonious and beneficial relationship that serves the best interests of both the educational institution and the families involved.

3.2 Strategy Space

To enhance the quality and efficiency of after-school services, a comprehensive analysis of the strategies is necessary. Schools and families have two strategic choices in the process of participating in after-school services: participating in after-school services or not participating in after-school services. For ease of expression, let participants in the game be referred to as School (S) and Family (F). Both schools and families have two strategic choices: Participate in after-school services (P) or Do not participate in after-school services (N). Specifically, it includes the following four scenarios:

- 1) (S-P, F-P): Both the school and the family opt to engage in after-school services, indicating a mutual agreement and participation that could lead to enhanced educational outcomes and resource utilization.
- 2) (S-P, F-N): The school opts to provide after-school services, but the family decides against participating. This situation might arise from a lack of perceived need or dissatisfaction with the services offered,

leading to under-utilization of the school's resources.

- 3) (S-N, F-P): The school decides not to offer after-school services, yet the family seeks to engage in such activities, potentially through external providers or self-organized initiatives. This scenario reflects a disconnect between the school's offerings and the family's desires, suggesting a need for alternative solutions.
- 4) (S-N, F-N): Neither the school nor the family chooses to engage in after-school services. This outcome could be due to various reasons, such as lack of funding, lack of interest, or other priorities taking precedence, resulting in no after-school activities being provided or sought.

Therefore, the strategic choices available to both parties are {(S-P, F-P), (S-P, F-N), (S-N, F-P), (S-N, F-N)}.

3.3 Utility Function and Payment Matrix

Under different strategy combinations, the benefits for schools and families vary. The utility function is shaped by a multitude of factors, necessitating a comprehensive approach [10]. Based on some conclusions we have obtained from our survey, it must take into account a range of factors including educational outcomes, cost burden, and resource utilization efficiency. For instance, if both parties participate in after-school services, it may lead to improved educational outcomes and effective resource utilization, but it could also increase cost burdens; if one party participates and the other does not, it may result in uneven resource distribution or a reduction in educational effectiveness. To simplify the analysis, we assume that the utility function mainly considers the following factors: educational outcomes (E), cost burden (C), resource utilization efficiency (R), and parental satisfaction (S). The specific values are examples, and actual situations need to be determined based on actual surveys.

Next, we assign utility values to each strategy combination and obtain the following payment matrix.

Table 1. Strategy choices and payoffs

Family (F)	Participate (P)	Not Participate(N)
School (S)		
Participate (P)	(a, b)	(c, d)
Not Participate (N)	(e, f)	(g, h)

In Table 1, *a, b, c, d, e, f, g, h* represent the benefits for schools and families under different strategy combinations. However, due to the complexity and multi-dimensionality of these benefits, it is challenging to directly calculate them using simple mathematical formulas, see [11,12]. In the following subsection, a qualitative description of the benefits will be provided, based on the basic principles of game theory and actual conditions.

3.4 Static Game Analysis

In static game analysis, it is assumed that schools and families have complete information and make decisions simultaneously. By analyzing the payoffs for both parties under different strategy combinations and identifying the Nash equilibrium [13,14]. The Nash equilibrium is a state where each participant has chosen the optimal strategy given the strategies of the other participants. Based on the aforementioned four factors *E, C, R, and S*, we analyze and predict the strategic choices and corresponding payoffs of both parties under four different cases.

- (1) (S-P, F-P), i.e., both parties participate in after-school services. In such case, schools are expected to have high educational outcomes, and families are expected to receive high educational benefits from after-school services. The resources (such as facilities, materials, and teacher time) are being used effectively and to a great extent when both parties are engaged in after-school services. The quality of services, the convenience of the program, and the perceived educational value for their children contribute to high satisfaction levels among families. Due to cost-sharing or subsidies, the financial impact is less heavy, resulting in a moderate cost burden. The overall payoff or utility is high when schools and families both participate in after-school services. The after-school services, particularly in rural areas, present a medium cost burden for families, a factor that, while not excessively high, remains significant in their decision-making process. This indicates that the benefits outweigh the costs, leading to a positive outcome for both parties. Thus the school's and the family's payoff are $E_{high} + R_{high} - C_{medium} = \text{high}$ and $E_{high} + S_{high} - C_{medium} = \text{high}$, respectively.

- (2) (S-P, F-N), i.e., the school participates but the family does not. In such case, Thus the school's and the family's payoff are $E_{medium} + R_{low} - C_{medium} = \text{low}$ and $E_{none} + S_{low} - C_{none} = \text{low}$, respectively.

For the school, the medium educational outcome (E_{medium}) suggests that the school is providing after-school services but the impact on students' learning or development is not as high as it could be. This could be due to a lack of resources, inadequate program planning, or insufficient teacher training. The low resource utilization (R_{low}) indicates that the school's facilities, materials, or staff time are not being used to their full potential, possibly because the program is not well-attended or there is a mismatch between available resources and student needs. The medium cost (C_{medium}) implies that there are still significant expenses associated with running the program, which could be due to fixed costs like facility maintenance or staffing, even if the program is not operating at full capacity. When these elements are combined, the school's overall payoff is low because the benefits of providing the service (medium educational outcomes and low resource utilization) do not outweigh the costs.

For the family, the family is not participating in the after-school service, and thus the absence of educational outcomes (E_{none}) means that the family is not participating in the after-school service, and thus, there are no academic benefits for their child. The low satisfaction (S_{low}) reflects the family's discontent with the situation. This could be because they are unable to access the services due to cost, location, or other barriers, and as a result, they are not satisfied with the options available to support their child's after-school needs. The zero cost (C_{none}) indicates that since the family is not participating, there are no direct financial costs for them. However, this does not account for potential opportunity costs or the value of time and convenience they might be missing out on. The family's overall payoff is low because they are not engaged in the after-school services, which leads to no educational benefits and low satisfaction, despite there being no direct financial burden.

- (3) (S-N, F-P), i.e., the school does not participate but the family seeks alternative means to provide after-school services. Thus the school's and the family's payoff are $E_{none} + R_{none} - C_{none} = \text{none}$ and $E_{medium} + S_{medium} - C_{high} = \text{medium}$, respectively.

For the school, the absence of educational outcomes (E_{none}) and resource use (R_{none}) indicates that the school does not participate in providing after-school services. This could be due to various reasons such as lack of funding, policy constraints, or a strategic decision to not offer such services. The zero cost (C_{none}) reflects that since the school is not offering after-school services, there are no direct financial implications for the school. However, this does not consider potential indirect costs like missed opportunities for student engagement or community outreach.

For the family, the medium educational outcomes (E_{medium}) suggest that the family seeks and finds alternative means to provide after-school services for their child, which meets some but not all of their educational expectations. The quality might be compromised due to non-professional services. Medium satisfaction (S_{medium}) might be due to the adequacy of the alternative services found, but this satisfaction could be tempered by concerns about the quality or suitability of these services compared to what might be offered by the school. The high cost (C_{high}) stems from the family's need to both find and pay for these alternatives. This financial burden is significant and could be a limiting factor for some families.

- (4) (S-N, F-N), i.e., neither party participates in after-school services.

For the school, it does not offer after-school services, and thus, there is no academic benefit for students, the educational outcomes is absent (E_{none}). The school's facilities and staff time are not being used for after-school programs, which could be due to a lack of demand, insufficient resources, or a decision not to invest in such services, hence the absence of resource utilization, denoted by R_{none} . Consequently, there are no financial expenditures for the school in relation to after-school services, as they are not being provided, indicating that the cost variable is also null, denoted by C_{none} . Thus, the school's payoff can be calculated by $E_{none} + R_{none} - C_{none} = none$.

For the family, children can not access any after-school services, which could lead to unmet

educational needs for outside of regular school hours. Due to the lack of available after-school services that could provide supervision, tutoring, or enrichment activities for their children, the satisfaction of parent is low, denoted by S_{low} . This dissatisfaction is compounded if there is a perceived need for such services that is not being fulfilled. Certainly, the family is not incurring any direct financial expenses related to after-school services. Thus, the family's payoff is $E_{none} + S_{low} - C_{none} = low$.

Having established the assumptions of complete information and simultaneous decision-making in our static game analysis, we now illustrate in Table 2 the strategic choices and their respective payoffs. This table not only provides a clear view of the potential outcomes but also aids in identifying the Nash equilibrium.

As we can see from Table 2, the first strategy yields the highest benefit for the school and the family's benefit is also maximized by the school's choice, then (S-P, F-P) constitutes a Nash equilibrium.

3.5 Dynamic Gme Analysis

Considering the potential information asymmetry or sequential decision-making in practice, dynamic game analysis becomes more complex.

Firstly, information may not be fully transparent in practice. Schools may need to reduce information asymmetry and increase families' trust in the school's commitments through publicity and communication. Secondly, as the first mover, the school's decisions and actions will affect the family's response. In turn, the family's decisions will also affect the school's subsequent actions. This is a continuous interactive process. Thirdly, as the game progresses, both schools and families may adjust their strategies based on each other's reactions and changes in the market. Schools may optimize service content and financial support based on the participation and feedback from families. Finally, when formulating strategies, schools need to balance long-term goals (such as establishing a good reputation and attracting

Table 2. Strategy choices and payoffs

		Family (F)	
		Participate (P)	Not Participate (N)
School (S)	Participate (P)	(high, high)	(low, low)
	Not Participate (N)	(none, medium)	(none, low)

more users) with short-term actions (such as immediately attracting family participation). Families also need to consider short-term benefits (such as their children's immediate learning needs) and long-term benefits (such as their children's long-term educational development).

In dynamic game analysis, the decisions and actions of schools and families are interdependent and will adjust with changes in time and information. The following is the process of the dynamic evolution of decisions for both parties:

- (1) Initial phase: Schools, as the first movers in the game, announce their commitment to providing high-quality after-school services and pledge to reduce the financial burden on families. This strategy aims to attract families who were initially hesitant to participate in after-school services by sending a positive signal. The school's action is based on an anticipation of market demand (i.e., families' demand for after-school services) and attempts to create a competitive edge by promising high-quality services and financial support.
- (2) Family response phase: After observing the school's actions, families will make decisions based on the school's promises, their own needs, and financial situation. If the school's promises are deemed credible, and families believe that participating in after-school services will bring tangible benefits (such as academic progress for children and peace of mind for working parents), then families are likely to choose to participate. This choice is not only an endorsement of the school's promises but also a balance of their own interests.
- (3) School's follow-up actions: Once families generally show a positive attitude towards after-school services, schools will be further motivated to invest more resources to improve the services. This is because the school's goal is to maximize participation rates and service quality to maintain its reputation and attract more potential users. The positive response from families provides positive market feedback for the school, prompting the school to continue to increase investment, forming a virtuous cycle.

4. CONCLUSIONS

Combining the results of game analysis and survey research, we can draw the following summary analysis:

1. Schools and families have shown a interactive relationship in the provision and participation of after-school services. The school's proactive strategy, which offers high-quality services and financial support, has successfully attracted family participation.
2. Accurate anticipation and timely response to family needs are crucial to the success of the school's strategy. By providing a variety of after-school services, schools can meet the diverse needs of families, thereby enhancing their market competitiveness.
3. The research findings show that effective resource utilization and continuous optimization of service quality are key to improving after-school services. Schools have enhanced the attractiveness of their services and increased the job satisfaction and happiness of teachers by organizing them to learn new skills and carry out special projects.
4. In urban schools, the limitation of spatial resources has become the main factor restricting the diversity of after-school services. Although there is an abundance of teaching resources, the lack of sufficient activity venues and facilities limits the richness of service content, affecting the satisfaction of students and parents.
5. The current state of after-school services in rural primary and secondary schools shows a clear trend of polarization. On one hand, some schools have successfully improved service quality and participation through innovation and effort; on the other hand, the limitations of resources and teaching staff have led to a year-by-year decline in service quality and participation in some schools.
6. Whether in rural or urban areas, continuous improvement of the quality and diversity of after-school services is key to enhancing the overall level of educational services. Schools need to constantly adjust and optimize their service strategies to adapt to the constantly changing market demands and family expectations.
7. Policy makers and communities should provide more support to help schools

overcome the limitations of resources and space, and improve the quality and accessibility of after-school services. Through policy guidance and community participation, balanced development of educational services can be promoted to meet the educational needs of more families.

When conducting game analysis, we must consider both schools and families, as well as broader societal factors. Policy direction, economic conditions, and cultural background are key societal factors that can significantly influence participation and the effectiveness of after-school services. Moreover, the research process has revealed other factors that must be considered, especially for students in rural areas. For instance, during the winter season, students who live farther away may find it dark when they return home after participating in after-school services. This not only increases the safety risks for students but may also affect parents' attitudes and choices regarding after-school services. These practical issues need to be fully considered in policy formulation and school service planning to ensure that after-school services can meet educational needs while also safeguarding student safety and providing convenience for families.

DISCLAIMER (ARTIFICIAL INTELLIGENCE)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of manuscripts.

ACKNOWLEDGEMENT

This study was funded by the Scientific Research Funds of Hunan Provincial Education Department (Grant number 23C0381).

COMPETING INTERESTS

The authors declare that they have no competing interests exist.

REFERENCES

1. Xu Q. The dilemma and implementation strategies of after-school service execution in primary and secondary schools from the perspective of stakeholders. *Journal of*

- Nanjing Xiaozhuang University. 2024;40(03):64-71.
2. Long B. and Li S. A Study on the Collaborative Supply Mechanism of School After-School Services from the Perspective of Actor-Network Theory. *Educational Science*. 2024;40(01):36-42.
 3. Duan J, Li R, Hui J. Analysis of the cooperative game between government and social forces in the supply of after-school sports services. *Bulletin of Sports Science and Technology*. 2023;31(06):87-90+100.
 4. Sun F. Research on the problems and collaborative governance of after-school services in primary and secondary schools in T city under the background of double reduction. *Shandong University of Finance and Economics*; 2024.
 5. Li M. A Study on the Implementation and Improvement of Junior High School After-School Services from the Perspective of Stakeholders. East China Normal University; 2023.
 6. He J, Yu H, Jiang M, Szumilewicz A. Physical activity programs in shanxi province schools in china: effects of in-school and after-school delivery on students' motivational and social outcomes. *Sustainability*. 2023;15(10): 8080
 7. Song J, Liu J, Xie L. An empirical study on the satisfaction model of after-school services for elementary school students after the double reduction policy. *Modern Primary and Secondary School Education*. 2024;40(06): 6-11.
 8. Wang W. and Liu C. A survey report: An analysis of the preliminary effect of the implementation of double reduction. *Journal of Global Research in Education and Social Science*. 2022;16(2): 1-6.
 9. Qiu Q. and Liu C. Implementation status and strategies of double reduction policy and after-school services in rural primary and secondary schools. *Educational Progress*. 2023;13(10):7991-7997.
 10. Wu Z, Hu L. Research on game industry cooperation based on evolutionary game model, *Journal of Mathematics*. 2023;2023: 9967797.
 11. Song J, Liu J, Xie L. An Empirical study on the satisfaction model of after-school services for elementary school students after the 'double reduction' policy. *Modern Primary and Secondary School Education*. 2024;40(06):6-11.

12. Duan J. A study on parent satisfaction of after-school services in primary schools in Xingyang City. Zhengzhou University; 2021.
13. Holt CA, Roth AE. The nash equilibrium: A Perspective. Proceedings of the National Academy of Sciences. 2004;101(12):3999-4002.
14. Nash JF. Equilibrium points in N- Person games. Proceedings of the national academy of sciences, 1950;36(1): 48- 49.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of the publisher and/or the editor(s). This publisher and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.

© Copyright (2024): Author(s). The licensee is the journal publisher. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:

The peer review history for this paper can be accessed here:

<https://prh.ikpress.org/review-history/12298>