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Time-Frequency Coherence and Forecast Analysis of Selected Stock Returns in Ghana Using Haar Wavelet

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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Abstract

Aims/ objectives: The study seeks to analyze the correlation of some selected stock returns with respect to both time and frequency domain, and also to forecast returns using Wavelet Coherence and Wavelet-ARIMA model as alternative to Pearson correlation and ARIMA model respectively.

Study Design: Financial Mathematics.

Place and Duration of Study: August 2016 to July 2017 , Department of Mathematics, Kwame Nkrumah University of Science and Technology.

Methodology: We transform data using the Haar Wavelet as the basis function.

Results: Results revealed interesting dynamics of correlations altering in time and across frequencies continually between paired returns. Furthermore, Wavelet-Arima method was found to be more appropriate for forecast with minimal error measure of forecast values.

Conclusion: Given the heterogeneous trading behavior in stock markets, investors operate at different frequencies for their trade and investment preferences. Thus, apart from the time domain, there is a frequency domain, which represents various investment horizons.

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Keywords: Co-movement; stock returns; wavelet coherence.

1 Introduction

Many years have been dedicated to searching for new ways to improve traditional techniques of analyzing changes in the structure of time series. A recent possible approach for solving this task is a detailed analysis of data from past periods. In this field, several methods are used; the most well-known are spectral analysis, trend analysis and autocorrelation analysis. [1]. The interconnection between stock markets have grown significantly over the past decade. As a result, the Ghanaian stock market has become more open to foreign investors. In financial markets with growing trading volumes and liquidity, the integration and co-movements are becoming stronger in time. Over the years, time series analysis are mostly done in time or spectral domain separately, with the assumption that data is stationary and market activities are stable across all periods. This case is not always true since multi-scaling features are natural concept in econometric time series analysis in reference to an observed time series. These time series may contain several structures with their occurrences on different time scales, [2],[3]. Classical correlation models give overall variance which makes it difficult to figure out the inter-relation between stocks at diverse time horizons,[4]. The wavelet approach is more flexible since it can be applied to both stationary and non-stationary time series thus preserving entirely the main driving forces or processes contained inside the data. The major advantage of wavelet method is to be applied without imposing any restriction to the signal [5]. Wavelets are treated as a 'lens' that enables the researcher to explore relationships that previously were unobservable. Thus we are able to uncover the interactions which can hardly be visible from any other modern econometric methods and which would stay hidden otherwise [6]. Realizing the relevance of time-scale structures on decision-making processes, research on the multi-resolution analysis of correlations in the stock market is become of great importance due to its changes in co-movement over time, [7]. Many literature confirm the interest of wavelet usage in financial analysis, recent studies include the works of [5], [8], [6].

In this work, we discuss the wavelet transform and wavelet coherence for examining relationships in both time frequency space between two time series. By studying wavelet power spectra and cross-wavelet transform, the results shows how correlations are altering in time and across frequencies continually.

The paper is structured as follows: a brief introduction to the methodology of the wavelet analysis in section 2, data analysis in section 3, and conclusions in section 4 .

2 Materials and Methods

In this section we discussed in brief the methodology employed in the research work. In wavelet analysis, we rely on wavelet power spectrum, cross-wavelet analysis, wavelet coherency and phase differences. The wavelet power spectrum demonstrates the volatilities and spikes in the data series; cross-wavelet analysis can be interpreted as co-variance of time series analysis; wavelet coherency can be interpreted as correlation in the time series analysis; and phase- difference provide the evidence of lead-lag relationship. In this paper, we focused on a description of the wavelet transform and coherence. These techniques provide powerful tools for analyzing time series.

2.1 Wavelet Transform

The Wavelet Transform can be defined as inner product of the time series F(t) and the basis functions $\psi(t)$ denoted as

$$W_x(s,\tau) = \langle F(t), \psi_{\tau,s}(t)\rangle = \int F(t)\psi^*_{s,\tau}(t)dt \tag{1}$$

These basis functions (wavelets) are generated from a single mother wavelet ($\phi(t)$) through a scaling parameter, 's' and translation parameter $'\tau'$ defined as:

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \tag{2}$$

The $s^{-1/2}$ factor is attributed to normalization of energy across all scales to ensure equal energy distributions at every scale as well as making the daughter wavelets comparable across scales and time.

The time series can be reconstructed from its weight transform given as

$$F(t) = \frac{1}{C_\psi}\int\int W(s,\tau)\psi_{s,\tau}(t)d\tau ds \tag{3}$$

C_ψ as the admissibility condition of wavelets.

$$C_\psi = \int_{-\infty}^{\infty}\frac{|\psi(\omega)|^2}{\omega}d\omega < \infty \tag{4}$$

$\psi(\omega) = \int_R \psi(t)e^{-i\omega t}dt$ represent the Fourier transform of $\psi(t)$

2.2 Discrete Wavelets Transform

Discrete wavelet transform (DWT) is any wavelet transform for which the wavelets are discretely sampled. Such wavelets are translated and scaled in discrete intervals of equation (2) and represented as [9].

$$\psi_{j,k}(t) = \frac{1}{\sqrt{s_0^j}}\psi\left(\frac{t-k\tau_0 s_0^j}{s_0^j}\right) \tag{5}$$

Where $j,k \in (Z)$, $s_0 > 1$ and $\tau > 0$

2.3 Haar Wavelet

Haar wavelet is the first discrete wavelet transform invented by Hungarian mathematician Alfred Haar (1909). It is also known to be the simplest transform of all existing piecewise constant wavelet functions. The orthogonal set of Haar functions are defined in the interval $[0,1]$ into scaling (h_0) and wavelet functions (h_i) respectively.

$$h_0(x) = \begin{cases} 1, & x \in [0,1) \\ 0, & otherwise \end{cases} \tag{6}$$

$$h_i(x) = \begin{cases} 1, & x \in [a,b) \\ -1, & x \in [b,c) \quad i = 1,2,\cdots \\ 0, & otherwise \end{cases} \tag{7}$$

$$a = \frac{k}{m} \qquad b = \frac{k+0.5}{m} \qquad c = \frac{k+1}{m}$$

$m = 2^j;\ \{j = 0,1,2,\cdots,J\} \qquad k = 0,1,2,\cdots,m-1 \qquad i = m+k+1$

The derivation for these coefficient values can be found in the works of [9] and [10].

2.4 Wavelet Power Spectrum (WPS)

The local wavelet power spectrum is defined as the absolute squared of the wavelet transform.

$$WPS_x(s,\tau) = |W_{x,\psi}(s,\tau)|^2 \tag{8}$$

The WPS depicts and measures the local variance of a signal at various scales s by giving information on the relative power (energy) at certain time and scale (frequency). Hence variance decomposition with a good time localization of the time series is done under investigation, [11].

2.5 Cross-Wavelet Transform(XWT) and Cross-Wavelet Power(XWP)

Given two series, X(t) and Y(t), the cross-wavelet transform (XWT) of these two time series which was first made known by [12]

is illustrated as

$$XWT = W_{x,\psi}W^{*}_{y,\psi} = W_{x,y} \tag{9}$$

With $W_{x,\psi}$ *and* $W_{y,\psi}$ as the series wavelet transforms respectively, and the cross-wavelet power is defined as ,

$$XWP_{x,y} = |W_{x,y}(s,\tau)| \tag{10}$$

Whiles the wavelet power spectrum depicts the series local variance, the cross-wavelet power of two time series characterize the local co-variance at each time and frequency.

2.6 Wavelet Coherency

Coherency is analogue of classical correlation. To identify both frequency-bands and time-intervals when two signals are related, Wavelet Coherency is used. Given signal $X(t), Y(t)$, their wavelet coherency is defined as,

$$R^2_{x,y}(s,\tau) = \frac{|S(s^{-1}W_{x,y}(s,\tau))|}{\sqrt{S(s^{-1}|W_{x,\psi}|^2) \times S(s^{-1}|W_{y,\psi}|^2)}},\ 0 \leq R_{x,y}(s,\tau) \leq 1 \tag{11}$$

S is smoothing operator in both time and scale and it is dependent on the choice of mother wavelet and scaling. The wavelet coherence coefficient is equal to 1 at all scales if smoothing is not applied, [12]. Values close to 0 indicate weak correlation while values close to 1 are an evidence of strong correlation.

2.7 Phase-differencing

Angle of coherency is called the phase difference also termed as the phase lead. Wavelet coherence phase differences is use to establish relationship between two time series as well as details about delays of oscillation (cycles) of the two examined time series. The Wavelet coherence phase difference is defined following [13] definition as,

$$\phi_{x,y}(\tau,s) = tan^{-1}\left(\frac{I\left\{S(W_{x,y}(s,\tau))\right\}}{R\left\{S(W_{x,y}(s,\tau))\right\}}\right);\ \phi_{x,y}(\tau,s) \in [-\pi,\pi] \tag{12}$$

I and R refers to both imaginary and real part of the wavelet function. S is again a smoothing parameter achieved through convolution in both time and scale parameters,[14].

Phase is indicated by arrows on the wavelet coherence plots. A zero phase difference means that the examined time series move together at a particular scale s. Arrows pointing to the right (left) when the time series are in-phase (anti-phase), i.e. positively (negatively) correlated. Arrow pointing up means that the first time series leads the second one by 90^o , arrow pointing down indicates that the second time series leads the first one by 90^o . Usually, we have a mixture of positions, for example, arrow pointing up and right means that the time series are in phase with the first times series leading the second one.

3 Results and Discussion

The data used in this study comprise daily stock closing prices from financial, industrial and manufacturing sectors. Considered for the respective sectors are Ghana Commercial Bank (GCB), Unilever Ghana (UNIL) and Produce Buying Company (PBC). The research data was collected over a period of 9years starting from January 2007 to December, 2015. The logarithm of the returns was computed to avoid overnight returns. This is a 1990 trading days sample data excluding major holidays in Ghana.

Stocks daily closing returns shows how well or poorly a stock performs on the stock market. It also a big deal for investors, financial institutions and other stakeholders on which they base their decisions on. Also Financial Institutions monitor's stock's closing returns to make decisions regarding their investment portfolios. A descriptive statistics including sample means, minimums, maximums, standard deviations, skewness and kurtosis is seen from Table 1.

Table 1: Descriptive statistic for stock's returns

Statistic	GCB(GHS)	PBC(GHS)	UNIL(GHS)
Min	-0.123481	-0.024491	-1.335001
Mean	0.000447	0.00060	0.000579
Max	0.092976	0.024293	1.337629
Skewness	-0.065259	-0.449947	0.029677
kurtosis	28.337951	15.803267	860.343053
sd	0.011150	0.003172	0.043903

From Table (1), UNIL had the minimum as well as the maximum daily returns, but on average, PBC had higher daily returns.

3.1 Pearson Correlation Test

A correlation test was carried out between series giving a preliminary insight into the existence of movement among the time series variables.

Table 2: Pearson correlation test

	GCB	PBC	UNIL
GCB	1	-0.04423203	0.01762214
PBC		1	0.008119096
UNIL			1

A linear trend exist between these pairs as showed in Table 2 with low correlations. GCB-PBC showed negative correlations which indicates that both sectors are in opposite trend over the applicable period, whiles GCB-UNIL and PBC-UNIL were positively correlated. Hence sectors can be said to share a common driving force.

3.2 Empirical Findings of the Wavelet application

3.2.1 Wavelet Power Spectrum(WPS) of Stock's Returns

Horizontal axis shows time, while vertical axis shows scale in days.

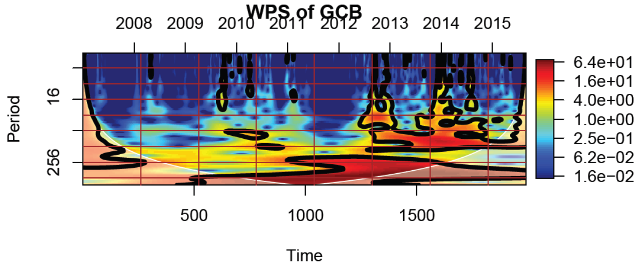

Figure 1: Wavelet power spectra of GCB daily stock returns

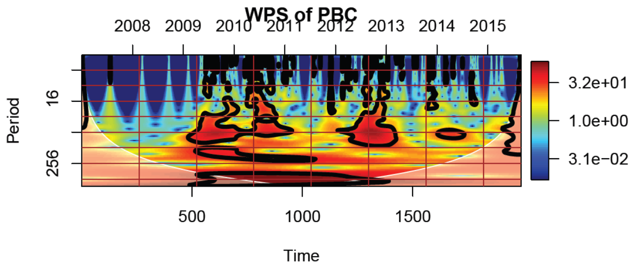

Figure 2: Wavelet power spectra of PBC daily stock returns

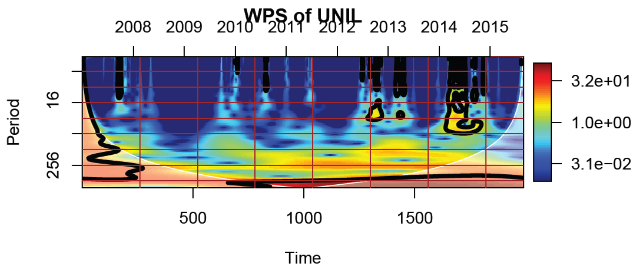

Figure 3: Wavelet power spectra of UNIL daily stock prices

Figure 1, 2 and 3 depicts the wavelet power of the three various stock returns. Looking at the time-scale decomposition of the stocks returns and the colors of the legend,variations are depicted by the color map to the right of the various figures. A High variation is showed for GCB through the period of 16 to 256 corresponding to time period, 2010-2015 from Figure 1. There is existence of much variation also for PBC depicted in Figure 2 through out the period mostly from 2009-2015. Lastly, UNIL experienced small rate of changes in their returns over all period as showed in Figure 3.. Comparatively, GCB and PBC index shows highest volatility while UNIL shows weaker volatility.

3.2.2 Wavelet Transform Coherency(WTC) of Stocks

We now concentrate on wavelet coherence which provides a picture on the co-movement of the analyzed stock markets. To asses the statistical significance, we use the Monte Carlo simulations. Time is on the horizontal axis while vertical axis refers to scale. The wavelet coherence finds the regions in time-scale space where the two time series co-vary (but do not necessarily have high power). Regions inside the black lines plotted in warmer colors represent regions where significant dependence has been found. The colder the color is the less dependent the series are. Thus the plot clearly identifies both frequency bands and time intervals where the series move together Returns data were decomposed using haar wavelet with 7 levels of decompositions.The wavelet scales are divided into 3 horizons: very short term of (2-4 days, 4-8 days, 8-16days), short term of (16-32 days, 32-64 days, 64-128 days) and long term of (128-256 days).

Figure 4 shows the wavelet coherence between GCB and PBC returns at different time scales. The graph shows that arrow directions at each frequency bands over the study duration is not the same. The study found that for very short holding periods consisting of (2-4days, 4-8days and 8-16days), there is low correlation between series along the observation period. For the short time horizon consisting of (16-32 days, 32-64 days, 64-128 days) holding periods, there is high correlation between returns in the year 2009 and 2012 to 2015. The phase difference shows positive correlation with GCB lagging for 2009 but became anti-phase (negatively correlated) with GCB leading through the years 2012-2015 .

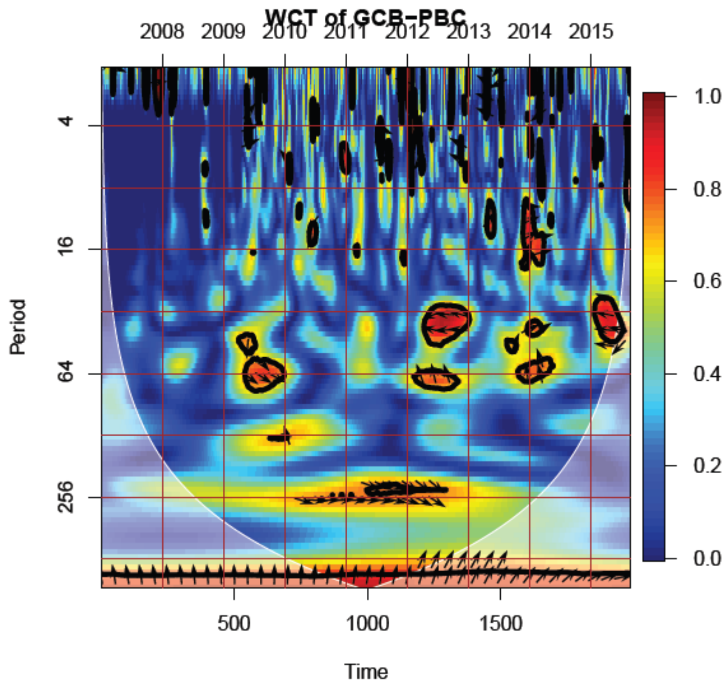

Figure 4: Wavelet coherence of GCB and PBC returns

Figure 5 shows the approximated wavelet coherence and the phase difference for GCB-UNIL pair . For very short investment horizon of (2-4days,4-8days and 8-16days) there is low correlation between returns along these observation periods. However, at the short investment horizon consisting of (16-32 days, 32-64 days, 64-128 days), the correlation between returns starts to be high and positive at ending of 2009 with GCB leading but became anti-phase at 2014 with GCB still leading. As the holding period becomes longer consisting of (128- 256 days) there is strong and positive correlation between returns from 2011 to 2013 with the lead variable as UNIL.

From Figure 6, at very short days (2-4 days, 4-8 days and 8-16 days)there is no clear significant

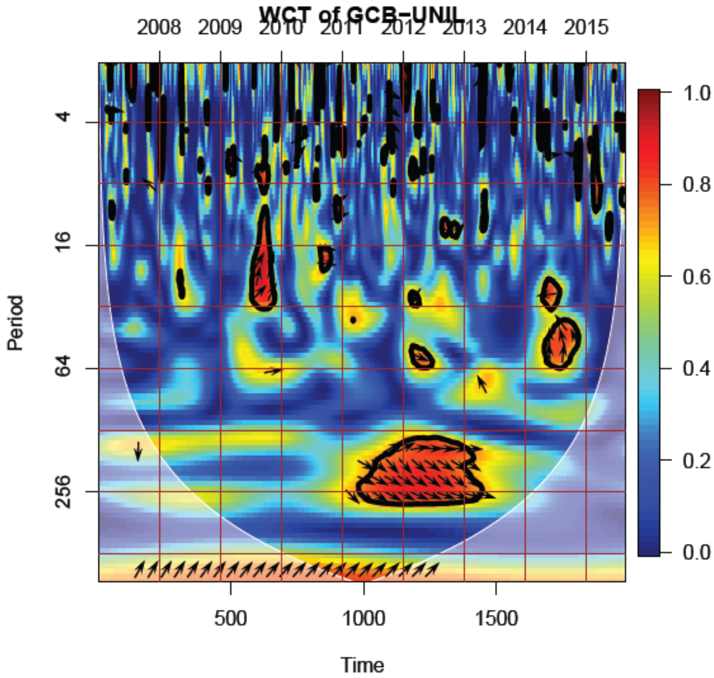

Figure 5: Wavelet coherence of GCB and UNIL retruns

co-movement,hence it can be said to have low correlation . For short terms (16-32 days,32-64 days and 64-128 days),there is high correlation (positively) at 2009 to 2010 with PBC lagging but became anti- phase (negative) at mid of 2010 with UNIL now leading. At long-terms of 128-256 returns are seen to be high and out of phases (negatively correlated) for 2013 to 2015 with UNIL leading (arrows slightly moving downwards).

3.2.3 Validation of Results-MODWT

Maximum Overlap Discrete Wavelet Transform (MODWT) approach is used for the validation of the results in Cross Wavelet Transform. In MODWT, decomposition of 7 scales (2-4, 4-8, 8-16, 16-32, 32-64, 64-128 and 128-256) all in days was done. The MODWT was used to examine the variance and correlations of GCB returns, PBC returns and UNIL returns at different time-scales.

Table 3 gives the coherence coefficient at each scale. GCB-PBC returns are negatively correlated from the very short term through to short term horizon. GCB-UNIL have negative correlations only at the short term horizon and PBC-UNIL returns are mostly positive correlated at various scales.

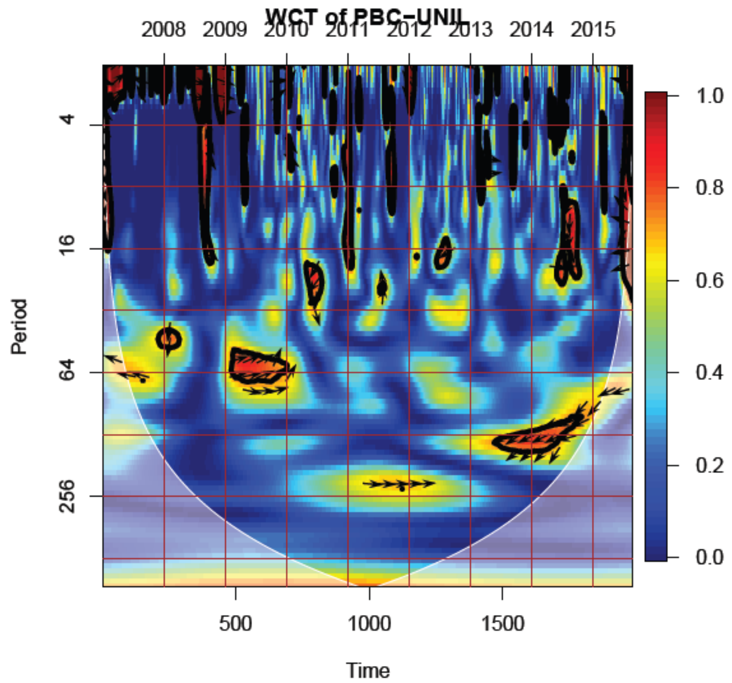

Figure 6: Wavelet coherency of UNIL and PBC returns

Table 3: Wavelet Correlation of stock returns at scales

modwt scale	GCB vs PBC	GCB vs UNIL	PBC vs UNIL
2-4 days	-0.06447225	0.0009201475	0.018197812
4-8 days	-0.02680940	0.0154357643	0.017557169
8-16 days	-0.02011260	0.0012102261	0.007077805
16-32 days	-0.04623706	-0.0018929673	-0.001389890
32-64 days	-0.14531928	-0.0457556548	0.049876221
64-128 days	0.02726640	0.1014480556	0.066456820
128-256 days	0.05674138	0.2425190861	0.053672965

3.3 Forecasting Stock Returns

Forecasting Approach
After the decomposition of the original series in detail sub-signals and an approximation sub-signal, ARIMA models are formulated for these new sub-signals.

Start = 1990 End = 1994 Frequency = 1
The ARIMA models were ARIMA(1,0,2) and Wavelet-ARIMA(1,0,2) for GCB, ARIMA(0,0,4) and Wavelet-ARIMA(0,0,4) for PBC and ARIMA(2,0,2) and Wavelet-ARIMA(2,0,2)for UNIL. The results of these methods are shown in Table 4 with their error measures in Table 5.

Table 4: forecast values of models for 5days-ahead

steps	ARIMA	Wavelet-ARIMA
	GCB	
1	0.0003089657	0.0002343983
2	0.0003301101	0.0002647556
3	0.0003402873	0.0002795341
4	0.0003495825	0.0002930262
5	0.0003580720	0.0003053439
	PBC	
steps	ARIMA	Wavelet-ARIMA
1	-5.310947e-05	-5.236242e-05
2	-1.574479e-04	-4.922182e-05
3	-1.447289e-04	-5.121904e-05
4	-8.974704e-05	-5.554755e-05
5	-6.024572e-05	-5.849261e-05
	UNIL	
steps	ARIMA	Wavelet-ARIMA
1	0.004113080	0.0010647794
2	0.001738571	0.0003060335
3	0.001667636	0.0002927731
4	0.001650000	0.0002962575
5	0.001633840	0.0003000608

Table 5: Forecast models comparison

Model	Stocks					
	GCB		PBC		UNIL	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
arima	0.0109259	0.004666086	0.003146249	0.001055658	00.03599144	0.008304951
wavelet(haar)-arima	**0.01076745**	**0.004536568**	**0.003100615**	**0.001025876**	**0.03547266**	**0.008115832**

4 Conclusions

The co-movements of stock returns and volatilities are important in asset allocation and risk management. In this paper, discrete wavelet analysis was employed to assess the correlations of Ghanaian stock market returns among three sectors. This methodology allows for the examination of the time and frequency varying correlation of stock markets within a unified framework.

The important implication of our findings was that, consistent with prior literature, the degree of stock market integration was changing over time, with a consistent changes in the pattern of the relationship for all stock market pairs at relatively higher frequencies from prior frequency levels. Viewing the phenomenon through portfolio diversification context, this implies that short term investors are interested in stock returns at higher frequencies, that is, short- term fluctuations and the long term investors are interested at lower frequencies, that is, long-term fluctuations.

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Competing Interests

Authors have declared that no competing interests exist.

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