



Some Inequalities for the Extension of k -Gamma Function

İnci Ege^{1*}

¹Department of Mathematics, Faculty of Art and Sciences, Aydın Adnan Menderes University, Aydın, Turkey.

Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/ARJOM/2019/v14i430133

Editor(s):

(1) Dr. Firdous Ahmad Shah, Associate Professor, Department of Mathematics, University of Kashmir, South Campus, India.

Reviewers:

(1) Snehadri Ota, Institute of Physics, India.

(2) Kwara Nantomah, University for Development Studies, Ghana.

(3) Li Yin, Binzhou University, China.

(4) Pasupuleti Venkata Siva Kumar, Vallurupalli Nageswara Rao Vignana Jyothi Institute of Engineering and Technology, India.

Complete Peer review History: <http://www.sdiarticle3.com/review-history/50206>

Received: 08 May 2019

Accepted: 17 July 2019

Published: 27 July 2019

Original Research Article

Abstract

In this paper, some inequalities involving the extension of k -gamma function are presented. Consequently, some previous results are recovered as particular cases of the present results.

Keywords: Gamma function; extension of k -gamma function; extension of k -digamma function; inequality.

2010 Mathematics Subject Classification: 33B15; 33B99; 26D07; 26A51.

1 Introduction

In recent years, some extensions of the well known Euler's classical gamma function have been considered by several authors. Also many properties and inequalities concerning these functions have been examined; see for example, [1], [2], [3], [4], [5], [6] and [7]. The Chaudhry-Zubair extension

*Corresponding author: E-mail: iege@adu.edu.tr;

of the gamma function is defined as [8]

$$\Gamma_b(z) = \int_0^\infty t^{z-1} e^{-t-\frac{t}{b}} dt, \quad \operatorname{Re}(b) > 0, \operatorname{Re}(z) > 0, \quad (1.1)$$

and satisfies the recursion relation and reflection formula respectively as

$$\begin{aligned} \Gamma_b(z+1) &= z\Gamma_b(z) + b\Gamma_b(z-1), \\ \Gamma_b(-z) &= b^{-z}\Gamma_b(z). \end{aligned}$$

In the case $b = 0$, The Chaudhry-Zubair extension of the gamma function conclude with the classical gamma function. Mubeen have introduced the following extension of k -gamma function [9]

$$\Gamma_{b,k}(z) = \int_0^\infty t^{z-1} e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt, \quad \operatorname{Re}(z) > 0, b \geq 0, k > 0. \quad (1.2)$$

Note that, when $b = 0$, $\Gamma_{b,k}(z)$ tends to the k -gamma function defined by [3]

$$\Gamma_k(z) = \int_0^\infty t^{z-1} e^{-\frac{t^k}{k}} dt, \quad k > 0, \operatorname{Re}(z) > 0. \quad (1.3)$$

Also, when $k = 1$, $\Gamma_{b,k}(z)$ tends to $\Gamma_b(z)$ and if both $b = 0$ and $k = 1$, then $\Gamma_{b,k}(z)$ tends to Euler's classical gamma function $\Gamma(z)$.

Some properties of the extended gamma k -function $\Gamma_{b,k}(x)$ are given in [9] as follows:

$$\Gamma_{b,k}(x+k) = x\Gamma_{b,k}(x) + b^k\Gamma_{b,k}(x-k), \quad b \geq 0, k > 0 \quad (\text{difference formula}), \quad (1.4)$$

$$b^x\Gamma_{b,k}(-x) = \Gamma_{b,k}(x), \quad \operatorname{Re}(b) > 0, k > 0 \quad (\text{reflection formula}). \quad (1.5)$$

Throughout of this work, \mathbb{N} indicates the set of natural numbers and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. By differentiating repeatedly (1.3) with respect to z , one can obtain

$$\Gamma_{b,k}^{(m)}(z) = \int_0^\infty t^{z-1} (\ln t)^m e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt, \quad \operatorname{Re}(z) > 0, b \geq 0, k > 0 \quad (1.6)$$

where $m \in \mathbb{N}$.

In this paper our goal is to give some inequalities concerning the function $\Gamma_{b,k}^{(m)}(x)$ for $x > 0$ by using similar techniques as in [10] and [11]. Our results are also generalizations of some known results in the literature.

2 Main Results

In this section we present our main results by using Hölder's, Minkowski's and Young's inequalities among other algebraic tools.

Lemma 1 ([12]). (*Hölder's Inequality*) Let $\alpha, \beta \in (0, 1)$ and $\alpha + \beta = 1$. If $f(x)$ and $g(x)$ are integrable real valued functions on $[0, \infty)$, then the inequality

$$\int_0^\infty |f(x)g(x)| dx \leq \left[\int_0^\infty |f(x)|^\alpha dx \right]^{\frac{1}{\alpha}} + \left[\int_0^\infty |g(x)|^\beta dx \right]^{\frac{1}{\beta}} \quad (2.1)$$

holds.

Theorem 1. Let $x, y > 0, b \geq 0, k > 0, \alpha, \beta \in (0, 1), \alpha + \beta = 1, m, n$ even, $m, n \in \mathbb{N}_0$ and $\alpha m + \beta n \in \mathbb{N}_0$. Then the extension of k -gamma function satisfies the inequality

$$\Gamma_{b,k}^{(\alpha m + \beta n)}(\alpha x + \beta y) \leq \left[\Gamma_{b,k}^{(m)}(x) \right]^\alpha \left[\Gamma_{b,k}^{(n)}(y) \right]^\beta.$$

Proof. By using the equation (1.6), we obtain

$$\Gamma_{b,k}^{(\alpha m + \beta n)}(\alpha x + \beta y) = \int_0^\infty t^{\alpha x + \beta y - 1} (\ln t)^{\alpha m + \beta n} e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt.$$

Then since $\alpha + \beta = 1$, and m, n are even we have

$$\begin{aligned} \Gamma_{b,k}^{(\alpha m + \beta n)}(\alpha x + \beta y) &= \int_0^\infty t^{\alpha(x-1)} (\ln t)^{\alpha m} e^{-\left(\frac{t^k}{k} - \frac{b^k t^{-k}}{k}\right)\alpha} t^{\beta(y-1)} (\ln t)^{\beta n} e^{-\left(\frac{t^k}{k} - \frac{b^k t^{-k}}{k}\right)\beta} dt \\ &\leq \left[\int_0^\infty t^{x-1} (\ln t)^m e^{-\left(\frac{t^k}{k} - \frac{b^k t^{-k}}{k}\right)} \right]^\alpha \left[\int_0^\infty t^{y-1} (\ln t)^n e^{-\left(\frac{t^k}{k} - \frac{b^k t^{-k}}{k}\right)} \right]^\beta, \end{aligned}$$

by using Hölder's inequality (2.1) and the result follows. □

Remark 1. By letting $k = 1$ in the Theorem 1, we obtain the Theorem 3.1 of [11].

The following definition is well known in the literature; see for example [13].

Definition 1. Let $f : [a, b] \subset \mathbb{R} \rightarrow (0, \infty)$. Then f is called a log-convex function, if

$$f(\alpha x + (1 - \alpha)y) \leq [f(x)]^\alpha [f(y)]^{1-\alpha}$$

holds for any $x, y \in [a, b]$ and $\alpha \in [0, 1]$.

Corollary 1. Let $x > 0, b \geq 0, k > 0, \alpha, \beta \in (0, 1), \alpha + \beta = 1, m$ even and $m \in \mathbb{N}_0$. Then the function $\Gamma_{b,k}^{(m)}(x)$ is log-convex.

Proof. From the Theorem 1 by letting $m = n$ we get

$$\Gamma_{b,k}^{(m)}(\alpha x + \beta y) \leq \left[\Gamma_{b,k}^{(m)}(x) \right]^\alpha \left[\Gamma_{b,k}^{(m)}(y) \right]^\beta,$$

which completes the proof. □

Corollary 2. Let $x > 0, b \geq 0$ and $k > 0$. Then the function $\Gamma_{b,k}(x)$ satisfies the inequality

$$\Gamma_{b,k}(x) \Gamma_{b,k}''(x) \geq [\Gamma_{b,k}'(x)]^2.$$

Proof. From the log-convexity property of $\Gamma_{b,k}(x)$ we have $[\ln \Gamma_{b,k}(x)]'' \geq 0$. Then

$$[\ln \Gamma_{b,k}(x)]'' = \frac{\Gamma_{b,k}'(x)}{\Gamma_{b,k}(x)} = \frac{\Gamma_{b,k}(x) \Gamma_{b,k}''(x) - [\Gamma_{b,k}'(x)]^2}{[\Gamma_{b,k}(x)]^2} \geq 0,$$

and the proof completes. □

Corollary 3. Let $x > 0, b \geq 0, k > 0, m \in \mathbb{N}_0$ and m even. Then the inequality

$$[\Gamma_{b,k}^{(m+1)}(x)]^2 \leq \Gamma_{b,k}^{(m)}(x) \Gamma_{b,k}^{(m+2)}(x)$$

holds.

Proof. Let $n = m + 2, \alpha = \beta = \frac{1}{2}$ and $x = y$ in the Theorem 1. □

Definition 2. We introduce the extended k -digamma (k -psi) function $\psi_{b,k}(x)$ as the logarithmic derivative of $\Gamma_{b,k}(x)$;

$$\psi_{b,k}(x) = \frac{d}{dx} \ln \Gamma_{b,k}(x) = \frac{\Gamma'_{b,k}(x)}{\Gamma_{b,k}(x)} = \frac{1}{\Gamma_{b,k}(x)} \int_0^\infty t^{x-1} \ln t e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt$$

and more generally the extended k -polygamma function $\psi_{b,k}^{(m)}(x)$ by

$$\psi_{b,k}^{(m)}(x) = \frac{d^{m+1}}{dx} \ln \Gamma_{b,k}(x)$$

for $b, k > 0, m = 1, 2, \dots$ and $x > 0$.

From the difference formula (1.4), we get

$$\ln \Gamma_{b,k}(x+k) = \ln x + \ln \Gamma_{b,k}(x) + k \ln b + \ln \Gamma_{b,k}(x-k).$$

Then,

$$\psi_{b,k}(x+k) = \frac{1}{x} + \psi_{b,k}(x) + \psi_{b,k}(x-k). \tag{2.2}$$

Theorem 2. The function $\psi_{b,k}(x)$ is increasing for $x > 0$.

Proof. Since $\Gamma_{b,k}(x)$ is log-convex function we have $[\ln \Gamma_{b,k}(x)]'' \geq 0$ for all $x > 0$. Then,

$$\psi'_{b,k}(x) = [\ln \Gamma_{b,k}(x)]'' = \frac{\Gamma_{b,k}(x)\Gamma''_{b,k}(x) - (\Gamma'_{b,k}(x))^2}{[\Gamma_{b,k}(x)]^2} \geq 0$$

by using the Corollary 2. □

Theorem 3. The following reflection formulas hold true for $b, k > 0, m = 1, 2, \dots$ and $x > 0$,

$$\psi_{b,k}(x) + \psi_{b,k}(-x) = \ln b, \tag{2.3}$$

$$\psi_{b,k}^{(m)}(x) = (-1)^{m+1} \psi_{b,k}^{(m)}(-x). \tag{2.4}$$

Proof. By using the reflection formula (1.5), we have

$$x \ln b + \ln \Gamma_{b,k}(-x) = \ln \Gamma_{b,k}(x),$$

and taking the derivative of both sides in the last equation, we obtain the equation (2.3). Also taking the derivatives of the equation (2.3) repeatedly, we get the equation (2.4). □

Theorem 4. Let $x, y > 0, b \geq 0, k > 0, m \in \mathbb{N}_0, m$ even, $\alpha, \beta \in (0, 1), \alpha + \beta = 1$ and $s \geq 0$. Then the inequality

$$\Gamma_{b,k}^{(m)}(\alpha x + \beta y + s) \leq [\Gamma_{b,k}^{(m)}(x + s)]^\alpha [\Gamma_{b,k}^{(m)}(y + s)]^\beta$$

is valid.

Proof. By using the equation (1.6) and Hölder's inequality, we have

$$\begin{aligned} \Gamma_{b,k}^{(m)}(\alpha x + \beta y + s) &= \int_0^\infty t^{(\alpha x + \beta y + s) - 1} (\ln t)^m e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \\ &= \int_0^\infty t^{\alpha x + s \alpha - \alpha} (\ln t)^{\alpha m} e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} \alpha t^{\beta y + s \beta - \beta} (\ln t)^{\beta m} e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} \beta \\ &\leq \left[\int_0^\infty t^{x + s - 1} (\ln t)^m e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \right]^\alpha \left[\int_0^\infty t^{y + s - 1} (\ln t)^m e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \right]^\beta \\ &= [\Gamma_{b,k}^{(m)}(x + s)]^\alpha [\Gamma_{b,k}^{(m)}(y + s)]^\beta. \end{aligned}$$

Hence;

$$\Gamma_{b,k}^{(m)}(\alpha x + \beta y + s) \leq [\Gamma_{b,k}^{(m)}(x + s)]^\alpha [\Gamma_{b,k}^{(m)}(y + s)]^\beta.$$

□

Theorem 5. Let $x, y > 0, b \geq 0, a, c, k > 0, m \in \mathbb{N}_0, m$ even, $\alpha, \beta \in (0, 1)$ and $\alpha + \beta = 1$. Then the function $\Gamma_{b,k}(x)$ satisfy the inequality

$$\Gamma_{b,k}^{(m)}(ax + cy) \leq [\Gamma_{b,k}^{(m)}(\frac{ax}{\alpha})]^\alpha [\Gamma_{b,k}^{(m)}(\frac{cy}{\beta})]^\beta.$$

Proof. Similarly, by the Hölder's inequality, we obtain

$$\begin{aligned} \Gamma_{b,k}^{(m)}(ax + cy) &= \int_0^\infty t^{ax+cy-1} (\ln t)^m e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \\ &= \int_0^\infty t^{ax-\alpha} (\ln t)^{\alpha m} e^{-\left(-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}\right)\alpha} t^{cy-\beta} (\ln t)^{\beta m} e^{-\left(-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}\right)\beta} dt \\ &\leq \left[\int_0^\infty t^{\frac{ax}{\alpha}-1} (\ln t)^m e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \right]^\alpha \left[\int_0^\infty t^{\frac{cy}{\beta}-1} (\ln t)^m e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \right]^\beta, \end{aligned}$$

establishing the result. □

Lemma 2. [12] (Young's Inequality) If a and b are nonnegative, $\alpha, \beta \in (0, 1)$ and $\alpha + \beta = 1$, then the inequality

$$a^\alpha b^\beta \leq \alpha a + \beta b \tag{2.5}$$

holds.

Corollary 4. Let $x, y > 0, b \geq 0, a, c, k > 0, m \in \mathbb{N}_0, m$ even, $\alpha, \beta \in (0, 1)$ and $\alpha + \beta = 1$. Then the following inequality holds

$$\Gamma_{b,k}^{(m)}(ax + cy) \leq \alpha \Gamma_{b,k}^{(m)}(\frac{ax}{\alpha}) + \beta \Gamma_{b,k}^{(m)}(\frac{cy}{\beta}).$$

Proof. The proof follows immediately by using the Theorem 5 and the Lemma 2.5. □

Remark 2. Let $m = n = 0$ and $a = b = 1$ in the Theorem 5. Then we obtain the Theorem 3.9 and Corollary 3.10 in [11].

Lemma 3. [12] (Minkowski's Inequality) Let $1 \leq p < \infty$. If $f(x)$ and $g(x)$ are integrable real valued functions on $[0, \infty)$, then the inequality

$$\left[\int_0^\infty |f(x) + g(x)|^p dx \right]^{\frac{1}{p}} \leq \left[\int_0^\infty |f(x)|^p dx \right]^{\frac{1}{p}} + \left[\int_0^\infty |g(x)|^p dx \right]^{\frac{1}{p}} \tag{2.6}$$

holds.

Theorem 6. Let $x, y > 0, b \geq 0, k > 0, m, n \in \mathbb{N}_0, m, n$ even, $\alpha, \beta \in (0, 1)$ and $u \geq 1$. Then the inequality

$$[\Gamma_{b,k}^{(m)}(x) + \Gamma_{b,k}^{(n)}(y)]^{\frac{1}{u}} \leq [\Gamma_{b,k}^{(m)}(x)]^{\frac{1}{u}} + [\Gamma_{b,k}^{(n)}(y)]^{\frac{1}{u}}$$

holds for $x, y > 0$.

Proof. Since $x^k + y^k \leq (x + y)^k$ for $x, y \geq 0$ and $k \geq 1$, by using Minkowski's inequality we obtain that

$$\begin{aligned} [\Gamma_{b,k}^{(m)}(x) + \Gamma_{b,k}^{(n)}(y)]^{\frac{1}{u}} &= \left[\int_0^\infty t^{x-1} (\ln t)^m e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt + \int_0^\infty t^{y-1} (\ln t)^n e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \right]^{\frac{1}{u}} \\ &\leq \left[\int_0^\infty \left(t^{\frac{x-1}{u}} (\ln t)^{\frac{m}{u}} e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} \right)^u dt + \int_0^\infty \left(t^{\frac{y-1}{u}} (\ln t)^{\frac{n}{u}} e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} \right)^u dt \right]^{\frac{1}{u}} \\ &\leq \left[\int_0^\infty t^{x-1} (\ln t)^m e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \right]^{\frac{1}{u}} + \left[\int_0^\infty t^{y-1} (\ln t)^n e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \right]^{\frac{1}{u}}, \end{aligned}$$

and the proof completes. □

Remark 3. By letting $k = 1$ in the Theorem 6, we obtain the Theorem 3.12 of [11].

Theorem 7. *The inequality*

$$\Gamma_{b,k}^{(m)}(x) \leq \frac{\Gamma_{b,k}^{(m-r)}(x) + \Gamma_{b,k}^{(m+r)}(x)}{2}$$

is valid for $x > 0$, $m, r \in \mathbb{N}_0$, m, r even such that $m \geq r$.

Proof. By direct computation, we obtain the result since we have

$$\begin{aligned} \Gamma_{b,k}^{(m-r)}(x) + \Gamma_{b,k}^{(m+r)}(x) - 2\Gamma_{b,k}^{(m)}(x) &= \int_0^\infty \left[\frac{1}{(\ln t)^r} + (\ln t)^r - 2 \right] (\ln t)^m t^{x-1} e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \\ &= \int_0^\infty [1 - (\ln t)^r]^2 (\ln t)^{m-r} t^{x-1} e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \geq 0. \end{aligned}$$

□

Theorem 8. *Let $b, k > 0$, $m \in \mathbb{N}_0$ and m even. Then for $0 < a \leq 1$, the inequalities*

$$[\Gamma_{b,k}^{(m)}(k)]^{a-1} \leq \frac{\Gamma_{b,k}^{(m)}(k+x)}{\Gamma_{b,k}^{(m)}(k+ax)} \leq \frac{\Gamma_{b,k}^{(m)}(2k)}{\Gamma_{b,k}^{(m)}(k+ak)} \tag{2.7}$$

hold true for $x \in [0, k]$. If $a \geq 1$, then the inequalities (2.7) are reversed.

Proof. From the Corollary 1 we have $\Gamma_{b,k}^{(m)}(x+k)$ is log-convex. Then logarithmic derivative of $\Gamma_{b,k}^{(m)}(x+k)$ is increasing. Let $f(x) = [\ln \Gamma_{b,k}^{(m)}(x+k)]'$ and $g(x) = \frac{[\Gamma_{b,k}^{(m)}(x+k)]^a}{\Gamma_{b,k}^{(m)}(ax+k)}$. Then

$$\ln g(x) = a \ln \Gamma_{b,k}^{(m)}(x+k) - \ln \Gamma_{b,k}^{(m)}(ax+k).$$

Now, taking derivatives of both sides of the last equation, we get

$$\frac{g'(x)}{g(x)} = a[f(x) - f(ax)].$$

If $0 < a \leq 1$ then $g'(x) \geq 0$, since $f(x)$ is increasing and $g(x) > 0$. Then the inequalities (2.7) follows for $x \in [0, k]$. Similarly, for $a \geq 1$ reverse of the inequalities (2.7) is satisfied. □

Theorem 9. *Suppose that $s \in (0, 1)$, $b, k > 0$, $m \in \mathbb{N}_0$ and m even. Then the inequality*

$$\Gamma_{b,k}^{(m)}(x+s) \leq [\Gamma_{b,k}^{(m)}(x)]^{1-s} [\Gamma_{b,k}^{(m)}(x+1)]^s$$

is valid for $x > 0$.

Proof. Let $f(t) = t^{(1-s)(x-1)}(\ln t)^{m(1-s)} e^{-(1-s)\left(\frac{t^k}{k} - \frac{b^k t^{-k}}{k}\right)}$ and $g(t) = t^{sx}(\ln t)^{ms} e^{-s\left(\frac{t^k}{k} - \frac{b^k t^{-k}}{k}\right)}$. Then by using Hölder's inequality we get

$$\begin{aligned} \Gamma_{b,k}^{(m)}(x+s) &\leq \left[\int_0^\infty \left(t^{(1-s)(x-1)}(\ln t)^{m(1-s)} e^{-(1-s)\left(\frac{t^k}{k} - \frac{b^k t^{-k}}{k}\right)} \right)^{\frac{1}{1-s}} dt \right]^{1-s} \times \\ &\quad \times \left[\int_0^\infty \left(t^{sx}(\ln t)^{ms} e^{-s\left(\frac{t^k}{k} - \frac{b^k t^{-k}}{k}\right)} \right)^{\frac{1}{s}} dt \right]^s \\ &= \left[\int_0^\infty t^{x-1}(\ln t)^m e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \right]^{1-s} \left[\int_0^\infty (t^x(\ln t)^m e^{-\frac{t^k}{k} - \frac{b^k t^{-k}}{k}} dt \right]^s \\ &= [\Gamma_{b,k}^{(m)}(x)]^{1-s} [\Gamma_{b,k}^{(m)}(x+1)]^s, \end{aligned}$$

completes the proof of the theorem. □

3 Conclusions

In this study, we establish some inequalities for the extension of k -gamma function by using the classical Hölder's and Minkowski's inequalities and other algebraic tools. The established results are generalizations of some previous results.

Acknowledgment

The author would like to thank the anonymous referees for careful reading of the manuscript.

Competing Interests

Authors has declared that no competing interests exist.

References

- [1] Akugre TA, Nantomah K. On some properties of the real-matrix variate gamma function. *International Journal of Mathematics and Its Applications*. 2019;7(2):97-103.
- [2] Askey R. The q -gamma and q -beta functions. *Appl. Anal.* 1978;8:125-141.
- [3] Díaz R, Pariguan E. On hypergeometric functions and pochhammer k -symbol. *Divulg. Mat.* 2007;15:179-192.
- [4] Díaz R, Teruel C. q, k -Generalized gamma and beta functions. *Journal of Nonlinear Math. Phys.* 2005;12(1):118-134.
- [5] Kokologiannaki CG, Krasniqi V. Some properties of the k -gamma function. *Le Matematiche*. 2013; LXVIII(1): 13-22.
- [6] Krasniqi V, Qi F. Complete monotonicity of a function involving the p -psi function and alternative proofs. *Glob. J. Math. Anal.* 2014;2(3):204-208.
- [7] Nantomah K, Prempeh E, Twum SB. On a (p, k) -analogue of the gamma function and some associated inequalities. *Moroccan Journal of Pure and Applied Analysis*. 2016;2(2):79-90.
- [8] Chaudhry MA, Qadir A, Rafique M, Zubair SM. Extension of Euler's beta function. *Journal of Computational and Applied Mathematics*. 1997;78:19-32.
- [9] Mubeen S, Purohit SD, Arshad M, Rahman G. Extension of k -gamma, k -beta functions and k -beta distribution. *Journal of Mathhematical Analysis*. 2016;7(5):118-131.

- [10] Nantomah K, Prempeh E, Twum SB. Inequalities involving derivatives of the (p, k) -gamma function. Konuralp Journal of Mathematics. 2017;5(1):232-239.
- [11] Atugba MA, Nantomah K. On some inequalities for the Chaudhry-Zubair extension of the gamma function. Asian Research Journal of Mathematics. 2019;14(1):1-9.
- [12] Mitrinović DS. Analytic inequalities. Springer-Verlag, New York; 1970.
- [13] Zhang X, Jiang W. Some properties of log-convex function and applications for the exponential function. Computers and Mathematics with Applications. 2012;63:1111-1116.

©2019 Ege; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:

The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)

<http://www.sdiarticle3.com/review-history/50206>