Journal of Advances in Mathematics and Computer Science

23(1): 1-11, 2017; Article no.JAMCS.34225

Previously known as British Journal of Mathematics & Computer Science
ISSN: 2231-0851

The 7 and 8 Families of Hybrid Block Methods for Numerical Solution of Initial Value Problems in Stiff Equations

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/JAMCS/2017/34225
Editor(s):
(1) Dr. Feyzi Basar, Department of Mathematics, Fatih University, Turkey.
Reviewers:
(1) G. Y. Sheu, Chang-Jung Christian University, Taiwan.
(2) Jorge F. Oliveira, School of Technology and Management Polytechnic Institute of Leiria, Portugal.
Complete Peer review History: http://www.sciencedomain.org/review-history/19722

Received: 19th May 2017
Accepted: 8th June 2017
Published: 27th June 2017

Original Research Article

Abstract

An independent hybrid block Simpson's methods with a very closely accurate members of order p=q+2 as a block was formulated. This was obtained through increasing the number k in the multi-step collocation (MC). Maple software was used to facilitate the derivation of the k-step continuous formulae for k=6 and 7. In this paper, one off-grid collocation point was added in the MC in-between the last two step sizes, to get the desirable schemes. These schemes were evaluated for simultaneous application on stiff equations. The numerical results obtained signified the efficiency of the schemes.

Keywords: Continuous Hybrid Block Schemes (CHBS); Multi-step Collocation (MC); ODEs; Stiff equations.

1 Introduction

The general linear methods were introduced to provide a unifying framework to study consistency, stability and convergence of the traditional methods. More recently, the use of hybrid block methods which compete successfully with other methods like Runge-Kutta and linear multi-step methods, see [1,2,3]. These methods for increasingly high orders, become very difficult to derive using invasion algorithm, and another approach has been sought using Maple and Matlap software programme.

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This paper is been classified into sections. In section 2.0 the MC procedure is constructed involving off-mesh collocation points for each k and we analyze on its convergence analysis obtained in a block form. We obtained the order and error constants in a block form, the stability regions are also plotted.

Section 3.0 is the numerical implementation of the block hybrid schemes on stiff (ODEs) and conclusion is given in section 4.0.

Definition 1.1 Linear Multi-Step Method [1].

A k-step linear multi-step (lmm) is of the form

$$\sum_{j=0}^{k}\alpha_j y_{n+j} = h\sum_{j=0}^{k}\beta_j f_{n+j} \tag{1.1}$$

Where

$$f_{n+j} = f(x_{n+j}, y_{n+j}), \quad y_{n+j} = y(x_{n+j})$$

α_j and β_j are constants and satisfy the constraints

$$\alpha_k \neq 0, \alpha_0^2 + \beta_0^2 > 0$$

(1.1) is explicit if $\beta_k = 0$ and implicit if $\beta_k \neq 0$

Definition 1.2 [4].

If a numerical method is forced to be used, in a certain interval of integration, a step length, which is excessively small in relation to the smoothness of the exact solution in that interval, then the problem is said to be stiff in that interval.

Unlike the linear definition of stiffness, the definition allows a single equation, not just a system of equations, to be stiff. It also allows a problem to be stiff in parts, a nonlinear problem may start off non-stiff and become stiff, or vice versa. It may even have alternating stiff and non-stiff internal.

2 Construction of the Methods

2.1 Derivation techniques of MC

Let us consider the first order system of ODEs

$$y^1 = f(x,y), \quad a < x < b, \quad y,f \in \Re^s \tag{2.1.1}$$

where y satisfies a given set of s associated conditions, which are either all initial, all boundary or mixed conditions. The idea of the k – step MC, following [3,4], is to find a polynomial U of the form

$$U(x) = \sum_{j=0}^{t-1} \phi_j(x)\, y_{n+j} + h \sum_{j=0}^{m-1} \varphi_j(x)\, f\left(x_j, u(\bar{x}_j)\right), \qquad x_n \le x \le x_{n+x} \tag{2.1.2}$$

Where t denotes the number of interpolation points x_{n+i}, $i = 0, 1, ---, t-1$, and m denote the number of distinct collocation points $\bar{x}_i \in [x_n, \; x_{n+k}]$, $i = 0,1,---, m-1$ *the points* $\bar{x}_i$ are chosen from the step x_{n+i} as well as one or more off – step points.

The following assumptions are made;

1. Although the step size can be variable, for simplicity in our presentation of the analysis in this paper, we assume it is constant $h = x_{n+1} - x_n$, $N = \dfrac{b-a}{h}$ with the steps given by $\{x_n / x_n = a + nh,\; n = 0,1,---, N\}$,
2. That (2.1.1) has a unique solution and the coefficients $\phi_j(x)$, $\varphi_j(x)$ in (2.1.2) can be represented by polynomials of the form

$$\phi_j(x) = \sum_{i=0}^{t+m-1} \phi_{j,i+1} x^i, \qquad j \varepsilon \{0,1,---, t-1\} \tag{2.1.3}$$

$$h\,\varphi_j(x) = \sum_{i=0}^{t+m-1} \varphi_{j,i+1} x^i \qquad j \varepsilon \{0,1,---, m-1\} \tag{2.1.4}$$

with constant coefficients $\phi_{j,i+1}$, $h\varphi_{j,i+1}$ to be determined using the interpolation and collocation conditions:

$$u(x_{n+i}) = y_{n+i},\; i\varepsilon \; \{0,1,---, t-1\} \tag{2.1.5}$$

$$u^1(\bar{x}_i) = f(\bar{x}_i, u(\bar{x}_j)), \;\; j\varepsilon\{0,1,---, m-1\} \tag{2.1.6}$$

With this assumptions we obtain an MC polynomial, following [4-9], in the form

$$u(x) = \sum_{i=0}^{t+m-1} a_i x^i, \quad a^i = \sum_{j=0}^{t-1} c_{i+1,j+1} + \sum_{j=0}^{m-1} c_{i+1,j+t+1} f_{n+j} \tag{2.1.7}$$

Where $x_n \le x \le x_{n+k}$ and $c_{ij}\; i, j\varepsilon\{1,2,---, t+m\}$ are constants given by the elements of the inverse matrix $C = D^{-1}$. The MC matrix D is a nonsingular (m+1) square matrix of the type

$$D=\begin{bmatrix} 1 & x_n & x_n^2 & - & - & - & x_n^{t+m-1} \\ 1 & x_{n+1} & x_{n+1}^2 & - & - & - & x_{n+1}^{t+m-1} \\ . & . & . & . & & & . \\ . & . & . & & . & & . \\ . & . & . & & & . & . \\ 1 & x_{n+t-1} & x_{n+t-1}^2 & - & - & - & x_{n+t-1}^{t+m-1} \\ 0 & 1 & 2\overline{x_0} & - & - & - & (t+m-1)\overline{x_0^{t+m-1}} \\ 0 & 1 & 2\overline{x_1} & - & - & - & (t+m-1)\overline{x_1^{t+m-1}} \\ . & . & . & . & & & . \\ . & . & . & & . & & . \\ . & . & . & & & . & . \\ 0 & 1 & 2\overline{x_{m-1}} & - & - & - & (t+m-1)\overline{x_{m-1}^{t+m-1}} \end{bmatrix} \quad (2.1.8)$$

2.2 Six steps block hybrid Simpson's method with one off-step point

The parameters required for equation (2.1.8) are k=6, t=1, m= k+2; (x_n , x_{n+1}) ,

$$\left(\overline{x}_0 = x_n\,,\ \overline{x}_1 = x_{n+1}\,,\ \overline{x}_2 = x_{n+2}\ ,\ \overline{x}_3 = x_{n+3}\ ,\ \overline{x}_4 = x_{n+4}\ ,\ \overline{x}_5 = x_{n+5}\ ,\ \overline{x}_{\frac{11}{2}} = x_{n+\frac{11}{2}}\ ,\ \overline{x}_6 = x_{n+6} \right)$$

Hence the matrix (2.1.8) takes the following shape.

$$D=\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 & 8x_n^7 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 & 8x_{n+1}^7 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 & 6x_{n+2}^5 & 7x_{n+2}^6 & 8x_{n+2}^7 \\ 0 & 1 & 2x_{n+3} & 3x_{n+3}^2 & 4x_{n+3}^3 & 5x_{n+3}^4 & 6x_{n+3}^5 & 7x_{n+3}^6 & 8x_{n+3}^7 \\ 0 & 1 & 2x_{n+4} & 3x_{n+4}^2 & 4x_{n+4}^3 & 5x_{n+4}^4 & 6x_{n+4}^5 & 7x_{n+4}^6 & 8x_{n+4}^7 \\ 0 & 1 & 2x_{n+5} & 3x_{n+5}^2 & 4x_{n+5}^3 & 5x_{n+5}^4 & 6x_{n+5}^5 & 7x_{n+5}^6 & 8x_{n+5}^7 \\ 0 & 1 & 2x_{n+\frac{11}{2}} & 3x^2_{n+\frac{11}{2}} & 4x^3_{n+\frac{11}{2}} & 5x^4_{n+\frac{11}{2}} & 6x^5_{n+\frac{11}{2}} & 7x^6_{n+\frac{11}{2}} & 8x^7_{n+\frac{11}{2}} \\ 0 & 1 & 2x_{n+6} & 3x_{n+6}^2 & 4x_{n+6}^3 & 5x_{n+6}^4 & 6x_{n+6}^5 & 7x_{n+6}^6 & 8x_{n+6}^7 \end{bmatrix} \quad (2.2.1)$$

Using the maple software environment to evaluate (2.2.1) at the grid points

$$x=x_{n+1}\,, x=x_{n+2}\,, x=x_{n+3}, x=x_{n+4},\ x = x_{n+5},\ \ x = x_{n+\frac{11}{2}},\ x = x_{n+6}$$

We obtain the seven discrete schemes, namely,

$$y_{n+1} = y_n + h\left[\frac{4553}{15120} f_n + \frac{107293}{90720} f_{n+1} - \frac{3727}{3360} f_{n+2} + \frac{19001}{15120} f_{n+3} - \frac{4271}{3780} f_{n+4} + \frac{3559}{3360} f_{n+5} - \frac{400}{567} f_{n+\frac{11}{2}} + \frac{4381}{30240} f_{n+6}\right]$$

$$y_{n+2} = y_n + h\left[\frac{4027}{13860} f_n + \frac{4454}{2835} f_{n+1} - \frac{103}{420} f_{n+2} + \frac{52}{63} f_{n+3} - \frac{3047}{3780} f_{n+4} + \frac{82}{105} f_{n+5} - \frac{16384}{31185} f_{n+\frac{11}{2}} + \frac{137}{1260} f_{n+6}\right]$$

$$y_{n+3} = y_n + h\left[\frac{361}{1232} f_n + \frac{345}{224} f_{n+1} + \frac{243}{1120} f_{n+2} + \frac{859}{560} f_{n+3} - \frac{36}{35} f_{n+4} + \frac{1053}{1120} f_{n+5} - \frac{48}{77} f_{n+\frac{11}{2}} + \frac{143}{1120} f_{n+6}\right]$$

$$y_{n+4} = y_n + h\left[\frac{3034}{10395} f_n + \frac{880}{567} f_{n+1} + \frac{16}{105} f_{n+2} + \frac{1952}{945} f_{n+3} - \frac{386}{945} f_{n+4} + \frac{16}{21} f_{n+5} - \frac{16384}{31185} f_{n+\frac{11}{2}} + \frac{104}{945} f_{n+6}\right]$$

$$y_{n+5} = y_n + h\left[\frac{295}{1008} f_n + \frac{28025}{18144} f_{n+1} + \frac{125}{672} f_{n+2} + \frac{1975}{1008} f_{n+3} + \frac{125}{756} f_{n+4} + \frac{955}{672} f_{n+5} - \frac{400}{567} f_{n+\frac{11}{2}} + \frac{275}{2016} f_{n+6}\right]$$

$$y_{n+\frac{11}{2}} = y_n + h\left[\frac{905773}{3096576} f_n + \frac{7180745}{4644864} f_{n+1} + \frac{310123}{1720320} f_{n+2} + \frac{7643933}{3870720} f_{n+3} + \frac{2019127}{15482880} f_{n+4} + \frac{1476079}{860160} f_{n+5} - \frac{4213}{9072} f_{n+\frac{11}{2}} + \frac{1925957}{15482880} f_{n+6}\right]$$

$$y_{n+6} = y_n + h\left[\frac{41}{140} f_n + \frac{54}{35} f_{n+1} + \frac{27}{140} f_{n+2} + \frac{68}{35} f_{n+3} + \frac{27}{140} f_{n+4} + \frac{54}{35} f_{n+5} + \frac{41}{140} f_{n+6}\right] \quad (2.2.2)$$

2.3 Seven steps block hybrid Simpson’s method with one off-step point

The parameters required for equation (2.1.8) are k=7, t=1, m= k+2; (x_n, x_{n+1}) :

$$\left(\bar{x}_0 = x_n,\ \bar{x}_1 = x_{n+1},\ \bar{x}_2 = x_{n+2},\ \bar{x}_3 = x_{n+3},\ \bar{x}_4 = x_{n+4},\ \bar{x}_5 = x_{n+5},\ \bar{x}_6 = x_{n+6},\ \bar{x}_{\frac{13}{2}} = x_{n+\frac{13}{2}},\ \bar{x}_7 = x_{n+7}\right)$$

Hence the matrix (2.1.8) takes the following shape.

$$D = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 & x_n^9 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 & 8x_n^7 & 9x_n^8 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 & 8x_{n+1}^7 & 9x_{n+1}^8 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 & 6x_{n+2}^5 & 7x_{n+2}^6 & 8x_{n+2}^7 & 9x_{n+2}^8 \\ 0 & 1 & 2x_{n+3} & 3x_{n+3}^2 & 4x_{n+3}^3 & 5x_{n+3}^4 & 6x_{n+3}^5 & 7x_{n+3}^6 & 8x_{n+3}^7 & 9x_{n+3}^8 \\ 0 & 1 & 2x_{n+4} & 3x_{n+4}^2 & 4x_{n+4}^3 & 5x_{n+4}^4 & 6x_{n+4}^5 & 7x_{n+4}^6 & 8x_{n+4}^7 & 9x_{n+4}^8 \\ 0 & 1 & 2x_{n+5} & 3x_{n+5}^2 & 4x_{n+5}^3 & 5x_{n+5}^4 & 6x_{n+5}^5 & 7x_{n+5}^6 & 8x_{n+5}^7 & 9x_{n+5}^8 \\ 0 & 1 & 2x_{n+6} & 3x_{n+6}^2 & 4x_{n+6}^3 & 5x_{n+6}^4 & 6x_{n+6}^5 & 7x_{n+6}^6 & 8x_{n+6}^7 & 9x_{n+6}^8 \\ 0 & 1 & 2x_{n+\frac{13}{2}} & 3x^2_{n+\frac{13}{2}} & 4x^3_{n+\frac{13}{2}} & 5x^4_{n+\frac{13}{2}} & 6x^5_{n+\frac{13}{2}} & 7x^6_{n+\frac{13}{2}} & 8x^7_{n+\frac{13}{2}} & 9x^8_{n+\frac{13}{2}} \\ 0 & 1 & 2x_{n+7} & 3x_{n+7}^2 & 4x_{n+7}^3 & 5x_{n+7}^4 & 6x_{n+7}^5 & 7x_{n+7}^6 & 8x_{n+7}^7 & 9x_{n+7}^8 \end{bmatrix} \quad (2.2.3)$$

Using the maple software environment to evaluate (2.2.3) at the grid points

$x = x_{n+1}, x = x_{n+2}, x = x_{n+3}, x = x_{n+4}, x = x_{n+5}, x = x_{n+6}, x = x_{n+\frac{13}{2}}, x = x_{n+7}$

We obtain the eight discrete schemes, namely,

$$y_{n+1} = y_n + h\left[\frac{6904181}{23587200} f_n + \frac{24976453}{19958400} f_{n+1} - \frac{7382233}{5443200} f_{n+2} + \frac{641023}{362880} f_{n+3} - \frac{3229573}{1814400} f_{n+4} + \frac{2523853}{1814400} f_{n+5} - \frac{2071633}{1814400} f_{n+6} + \frac{4345984}{6081075} f_{n+\frac{13}{2}} - \frac{35857}{259200} f_{n+7}\right]$$

$$y_{n+2} = y_n + h\left[\frac{209201}{737100} f_n + \frac{253552}{155925} f_{n+1} - \frac{75403}{170100} f_{n+2} + \frac{3502}{2835} f_{n+3} - \frac{75553}{56700} f_{n+4} + \frac{15172}{14175} f_{n+5} - \frac{50563}{56700} f_{n+6} + \frac{487424}{868725} f_{n+\frac{13}{2}} - \frac{1546}{14175} f_{n+7}\right]$$

$$y_{n+3} = y_n + h\left[\frac{83173}{291200} f_n + \frac{394389}{246400} f_{n+1} - \frac{3}{22400} f_{n+2} + \frac{8879}{4480} f_{n+3} - \frac{35829}{22400} f_{n+4} + \frac{27549}{22400} f_{n+5} - \frac{22529}{22400} f_{n+6} + \frac{15744}{25025} f_{n+\frac{13}{2}} - \frac{2727}{22400} f_{n+7}\right]$$

$$y_{n+4} = y_n + h\left[\frac{52498}{184275} f_n + \frac{250904}{155925} f_{n+1} - \frac{2264}{42525} f_{n+2} + \frac{7064}{2835} f_{n+3} - \frac{13574}{14175} f_{n+4} + \frac{15224}{14175} f_{n+5} - \frac{12944}{14175} f_{n+6} + \frac{3506176}{6081075} f_{n+\frac{13}{2}} - \frac{1592}{14175} f_{n+7}\right]$$

$$y_{n+5} = y_n + h\left[\frac{269245}{943488} f_n + \frac{1280525}{798336} f_{n+1} - \frac{5825}{217728} f_{n+2} + \frac{173875}{72576} f_{n+3} - \frac{27725}{72576} f_{n+4} + \frac{119765}{72576} f_{n+5} - \frac{76025}{72576} f_{n+6} + \frac{22400}{34749} f_{n+\frac{13}{2}} - \frac{8975}{72576} f_{n+7}\right]$$

$$y_{n+6} = y_n + h\left[\frac{2593}{9100} f_n + \frac{3096}{1925} f_{n+1} - \frac{33}{700} f_{n+2} + \frac{86}{35} f_{n+3} - \frac{369}{700} f_{n+4} + \frac{396}{175} f_{n+5} - \frac{299}{700} f_{n+6} + \frac{12288}{25025} f_{n+\frac{13}{2}} - \frac{18}{175} f_{n+7}\right]$$

$$y_{n+\frac{13}{2}} = y_n + h\left[\frac{132397603}{464486400} f_n + \frac{8212607897}{5109350400} f_{n+1} - \frac{59738627}{1393459200} f_{n+2} + \frac{227150027}{92897280} f_{n+3} - \frac{233609207}{464486400} f_{n+4} + \frac{1030790657}{464486400} f_{n+5} - \frac{57365867}{464486400} f_{n+6} + \frac{339677}{467775} f_{n+\frac{13}{2}} - \frac{52778531}{464486400} f_{n+7}\right]$$

$$y_{n+7} = y_n + h\left[\frac{959651}{3369600} f_n + \frac{4589683}{2851200} f_{n+1} - \frac{41503}{777600} f_{n+2} + \frac{128233}{51840} f_{n+3} - \frac{144403}{259200} f_{n+4} + \frac{597163}{259200} f_{n+5} - \frac{82663}{259200} f_{n+6} + \frac{1047424}{868725} f_{n+\frac{13}{2}} + \frac{13391}{259200} f_{n+7}\right] \quad (2.2.4)$$

2.4 The order and error constants of the block hybrid methods

The hybrid block methods which are obtained in a block form with the help of maple software have the following order and error constants for each case.

k=6 BHSM with one off-step point

The method k=6 is of order 8 as a block and has error constants

$$C_9 = \left(-\frac{209749}{29030400}, -\frac{653}{11340}, -\frac{2277}{35840}, -\frac{169}{28350}, -\frac{7325}{1161216}, -\frac{46301497}{731782400}, -\frac{9}{1400}\right)^T$$

The method k=7 is of order 9 as a block and has error constants

$$C_{10} = \left(\frac{13789}{2177280}, \frac{3541}{680400}, \frac{251}{44800}, \frac{457}{85050}, \frac{2425}{435456}, \frac{3}{560}, \frac{60323029}{11147673600}, \frac{8183}{1555200}\right)^T$$

2.5 Stability regions of the block hybrid Simpson's methods

To compute and plot the absolute stability regions of the block hybrid Simpson's methods, the methods are reformulated as general linear methods expressed as;

$$\begin{bmatrix} Y \\ -- \\ y_{i+1} \end{bmatrix} = \begin{bmatrix} A & / & U \\ - & - & - \\ B & / & V \end{bmatrix} \begin{bmatrix} hf(Y) \\ \\ y_{i-1} \end{bmatrix}$$

where,

$$A = \begin{bmatrix} a_{11} & . & . & . & a_{1s} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ a_{s1} & . & . & . & a_{ss} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & . & . & . & b_{1s} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ b_{k1} & . & . & . & b_{ks} \end{bmatrix}$$

$$Y = \begin{bmatrix} y_n \\ y_{n+1} \\ . \\ . \\ . \\ y_{n+k} \end{bmatrix} \qquad y_{i+1} = \begin{bmatrix} y_{n+k} \\ . \\ . \\ y_{n+k-1} \end{bmatrix} \qquad y_{i-1} = \begin{bmatrix} y_{n+k-1} \\ . \\ . \\ y_{n+k-2} \end{bmatrix}$$

and the elements of the matrices A,B,U and V are obtained from the interpolation and collocation and collocation points.

The elements of the matrices A, B, U and V are substituted into the stability matrix

$$M(z) = B_2 + zA_2(I - zA_1)^{-1}B_1 \qquad where \quad A_1 = A, A_2 = B, B_1 = U, B_2 = V$$

and the stability function

$$\rho(\eta, z) = \det(\eta I - M(z))$$

Computing the stability function with Maple yields the stability polynomial of the method which is plotted in Matlab to produce the required absolute stability region of the method.

2.5.1 Absolute stability region of the block hybrid method K=6

The block hybrid methods (2.2.2) with one off-grid point are arranged as shown below;

The coefficients of these methods expressed in tabular form bellow gives the coefficients of the new method.

The values of A, B, U and V into the stability matrix and the stability function are used in the Maple software to yields the stability polynomial of the hybrid block method.

Using a Matlab program, we obtained the stability region of the block hybrid Simpson's method for K= 6 as shown in Fig. 1.

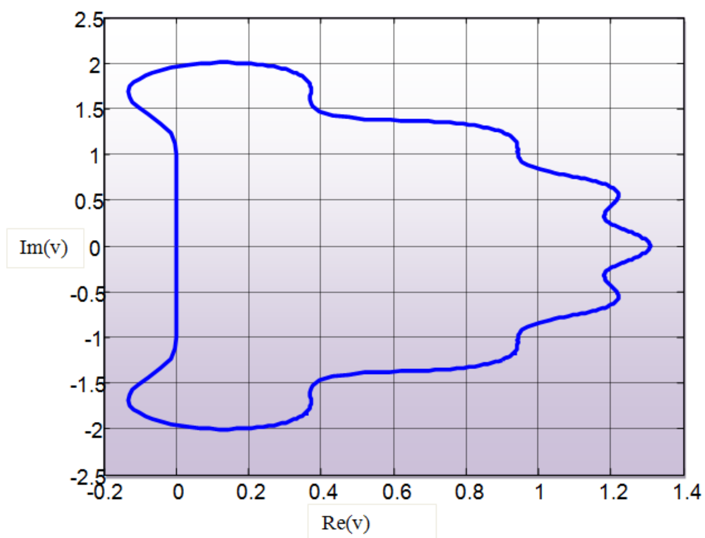

Fig. 1. Stability region of the block hybrid Simpson's method K=6

Following the same procedure for k=7, the elements of the matrices A,B,U and V are substituted and computing the stability function with Maple software yield, the stability polynomial of the method which is then plotted in MATLAB environment to produce the required absolute stability region of the methods, as shown by the Fig. 2.

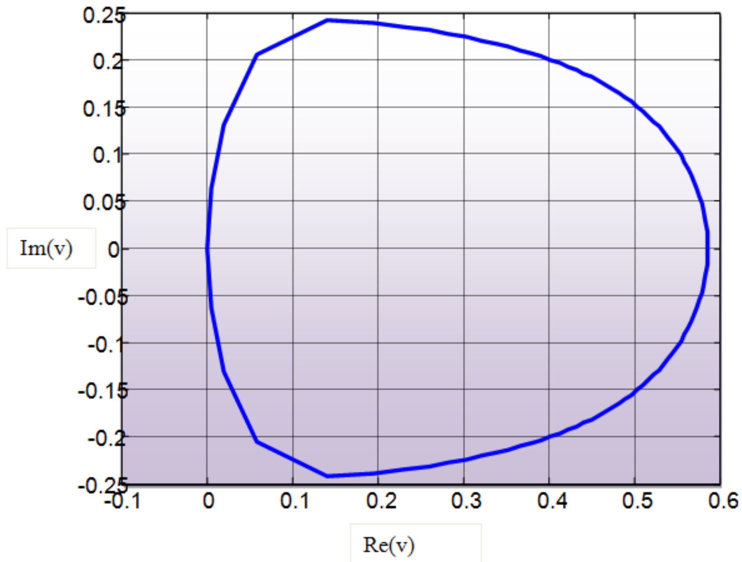

Fig. 2. Stability region of the block hybrid Simpson's method K=7

3 Numerical Implementation

To study the efficiency of the block hybrid method for k=6 and 7, we present some numerical examples widely used by several authors such as [2,5].

$$Experiment\ 1 \quad y^{1} = -1000000y, \quad where\ h = 0.1, \quad x \varepsilon [0,1]$$

$$Exact\ solution \quad y(x) = e^{-1000000x}$$

Experiment 2 $$y' = \begin{pmatrix} -21 & 19 & -20 \\ 19 & -21 & 20 \\ 40 & -40 & -40 \end{pmatrix} y, \ y(0) = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \quad 0 \le x \le 1, \ h = 0.1$$

$$y(x) = \tfrac{1}{2} \begin{pmatrix} e^{-2x} + e^{-40x}(\cos(40x) + \sin(40x)) \\ e^{-2x} - e^{-40x}(\cos(40x) + \sin(40x)) \\ 2e^{-40x}(\sin(40x) - \cos(40x)) \end{pmatrix}$$

Table 1. Absolute errors for experiment 1

Y	A-stable hybrid block Simpson's K=6	A-stable hybrid block Simpson's K=7
0.1	1.36×10^{-1}	1.21×10^{-1}
0.2	4.24×10^{-2}	3.30×10^{-2}
0.3	2.27×10^{-2}	1.54×10^{-2}
0.4	1.82×10^{-2}	1.20×10^{-2}
0.5	1.52×10^{-2}	1.20×10^{-2}

Y	A-stable hybrid block Simpson's K=6	A-stable hybrid block Simpson's K=7
0.6	9.09×10^{-2}	1.20×10^{-2}
0.7	1.24×10^{-2}	7.69×10^{-2}
0.8	3.86×10^{-3}	9.30×10^{-3}
0.9	2.07×10^{-3}	2.54×10^{-3}
1.0	1.65×10^{-3}	1.18×10^{-3}

Table 2. Absolute errors for experiment 2

Y	A-stable hybrid block Simpson's K=6			A-stable hybrid block Simpson's K=7		
	y_1	y_2	y_3	y_1	y_2	y_3
0.1	5.09×10^{-3}	5.09×10^{-3}	1.59×10^{-1}	2.27×10^{-3}	2.79×10^{-3}	1.44×10^{-1}
0.2	3.45×10^{-4}	3.45×10^{-4}	4.07×10^{-2}	5.56×10^{-4}	5.56×10^{-4}	3.23×10^{-2}
0.3	1.81×10^{-3}	1.81×10^{-3}	1.81×10^{-2}	9.46×10^{-4}	9.46×10^{-4}	1.33×10^{-2}
0.4	1.31×10^{-3}	1.31×10^{-3}	1.27×10^{-2}	9.08×10^{-4}	9.08×10^{-4}	7.69×10^{-3}
0.5	1.20×10^{-3}	1.20×10^{-3}	1.15×10^{-2}	1.02×10^{-3}	1.02×10^{-3}	6.91×10^{-3}
0.6	7.92×10^{-3}	7.92×10^{-3}	1.31×10^{-2}	1.10×10^{-3}	1.10×10^{-3}	7.54×10^{-3}
0.7	1.16×10^{-3}	1.16×10^{-3}	3.05×10^{-4}	6.60×10^{-3}	6.60×10^{-3}	9.84×10^{-3}
0.8	2.91×10^{-4}	2.91×10^{-4}	6.12×10^{-5}	8.44×10^{-4}	8.38×10^{-4}	1.23×10^{-4}
0.9	1.29×10^{-4}	1.29×10^{-4}	5.91×10^{-5}	1.83×10^{-4}	1.83×10^{-4}	6.31×10^{-5}
1.0	9.07×10^{-5}	9.07×10^{-5}	5.56×10^{-5}	7.01×10^{-5}	7.01×10^{-5}	4.27×10^{-5}

4 Conclusion

It is evident from the above tables that our proposed methods are indeed accurate, and can handle stiff equations. Also in terms of stability analysis, the methods, k=6 is A_α-stable and k=7 is A-stable and the schemes have also been shown to be of good order.

Competing Interests

Author has declared that no competing interests exist.

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