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Some Geometric Properties of a Non-Strict Eight Dimensional Walker Manifold

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

An 8−dimensional Walker manifold (M, g) is a strict walker manifold if we can choose a coordinate system $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ on (M,g) such that any function f on the manfold (M,g), $f(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = f(x_5, x_6, x_7, x_8)$. In this work, we define a Non-strict eight dimensional walker manifold as the one that we can choose the coordinate system such that for any f in (M, g), $f(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = f(x_1, x_2, x_3, x_4)$. We derive cononical form of the Levi-Civita connection, curvature operator, $(0, 4)$-curvature tansor, the Ricci tensor, Weyl tensor and study some of the properties associated with the class of Non-strict $8-dimensional$Walker manifold. We investigate the Einstein property and establish a theorem for the metric to be locally conformally flat.

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1 Introduction

Walker $2n-$manifold is a pseudo-Riemannian manifold which admit a non-trivial parallel null $r-$plane field with $r \leq n$. Walker 2n-manifold is applicable in physics. Lorentzian Walker manifolds have been studied extensively in physics since they constitute the background metric of the pp-wave models. A pp-wave spacetime admits a covariantly constant null vector field U [1].

The study of the curvature properties of a given class of Pseudo-Riemannian manifolds is important to our knowledge of these spaces. They are use to exemplify some of the main differences between the geometry of Riemannian manifolds and the geometry of pseudo-Riemannian manifolds and thereby illustrate phenomena in pseudo-Riemannian geometry that are quite different from those which occur in Riemannian geometry. See [2], [3],[4],[5],[6] for more on walker manifolds. The theory of Walker manifolds is outlined in [7]. The authors treated hypersurfaces with nilpotent shape operators, locally conformally flat metrics with nilpotent Ricci operator, degenerate pseudo-Riemannian homogeneous structures, para-Kaehler structures, and 2-step nilpotent Lie groups with degenerate center. The curvature properties of a large class of $4-$dimensional Walker metrics are treated in [8] and Several interesting examples are given. In particular as regards local symmetry, conformal flatness and Einstein-like metrics. There are several and interesting studies in pseudo-Riemannian manifolds. Some examples in this direction may be found in [9], [10], [11],[12], [13] [14] and references therein. Recall that a Walker metric is said to be Einstein Walker metric if its Ricci tensor is a scalar multiple of the metric at each point. $4-$dimensional Einstein Walker manifolds form the underling structure of many geometric and physical models such as; $hh-$space in general relativity, $pp-$wave model and other areas, for example, [14] and references therein. In [15], the geometric properties of some curvature tensors of an $8-$dimensional Walker manifold are investigated, theorems for the metric to be Einstein, locally conformally flat and for the $8-$dimensional manifold to admit a Kähler structure are given.

We want to extent this study to a canonical form for a Non-strict eight dimensional walker manifold. We derive the $(0, 4)$-curvature tansor, the Ricci tensor, Weyl tensor and study some of the properties associated with a class of Non-strict $8-dimensional$Walker manifold. We investigate the Einstein property and establish a theorem for the metric to be locally conformally flat.

A $2n-$dimensional pseudo-Riemannian manifold M admitting a parallel field of null $n-$dimensional planes D is given by the metric tensor:

$$\begin{pmatrix} 0 & Id_n \\ Id_n & B \end{pmatrix}$$

where Id_n is the $n \times n$ identity matrix and B is a symmetric $n \times n$ matrix whose entries are functions of the coordinates $(x_1, ..., x_{2n})$.

In particular, we want to work on an eight dimensional walker manifold M admitting a parallel field of null $4-$dimensional planes D given by the metric tensor:

$$g_{ij} = \begin{pmatrix} 0 & Id_4 \\ Id_4 & B \end{pmatrix} \tag{1.1}$$

Where 0 is a zero 4×4 matrix, Id_4 is a 4×4 identity matrix, and B is a 4×4 symmetric matrix whose coefficients are functions of $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$ defined as follows;

$$0 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, Id_n = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} a & b & 0 & 0 \\ b & 0 & 0 & 0 \\ 0 & 0 & b & c \\ 0 & 0 & c & 0 \end{pmatrix}$$

for an arbitrary smooth functions $a = a(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$, $b = b(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$, and $c = c(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$, defined on an open subset U of R^8.

We use the following equation:

$$\Gamma_{ij}^k = \sum_{l=1}^{8} \frac{1}{2} g^{kl}(\partial_i g_{jl} + \partial_j g_{il} - \partial_l g_{ij}), i, k, j = 1 - 8 \tag{1.2}$$

to obtain the non-vanishing components of the Christoffel symbols Γ_{ij}^k of the Levi-Civita connection of the Walker metric (1.1) as follows:

$$\begin{aligned}
&\Gamma_{55}^1 = \frac{a}{2}(\partial_1 a) + \frac{b}{2}(\partial_2 a) + \frac{1}{2}(\partial_5 a), \quad \Gamma_{55}^2 = \frac{b}{2}(\partial_1 a) + \frac{1}{2}(2\partial_5 b - \partial_6 a), \\
&\Gamma_{55}^3 = \frac{b}{2}(\partial_3 a) + \frac{c}{2}(\partial_4 a) + \frac{-1}{2}(\partial_7 a), \quad \Gamma_{55}^4 = \frac{c}{2}(\partial_3 a) + \frac{-1}{2}(\partial_8 a), \\
&\Gamma_{55}^5 = \frac{-1}{2}(\partial_1 a), \quad \Gamma_{55}^6 = \frac{-1}{2}(\partial_2 a), \quad \Gamma_{55}^7 = \frac{-1}{2}(\partial_3 a), \quad \Gamma_{55}^8 = \frac{-1}{2}(\partial_4 a), \\
&\Gamma_{56}^1 = \frac{a}{2}(\partial_1 b) + \frac{b}{2}(\partial_2 b) + \frac{1}{2}(\partial_6 a), \quad \Gamma_{56}^2 = \frac{b}{2}(\partial_1 b), \quad \Gamma_{56}^3 = \frac{b}{2}(\partial_3 b) + \frac{c}{2}(\partial_4 b) + \frac{-1}{2}(\partial_7 b), \\
&\Gamma_{56}^4 = \frac{c}{2}(\partial_3 b) + \frac{-1}{2}(\partial_8 b), \quad \Gamma_{56}^5 = \frac{-1}{2}(\partial_1 b), \quad \Gamma_{56}^6 = \frac{-1}{2}(\partial_2 b), \quad \Gamma_{56}^7 = \frac{-1}{2}(\partial_3 b), \\
&\Gamma_{56}^8 = \frac{-1}{2}(\partial_4 b), \quad \Gamma_{57}^1 = \frac{1}{2}(\partial_7 a), \quad \Gamma_{57}^2 = \frac{1}{2}(\partial_7 b), \quad \Gamma_{57}^3 = \frac{1}{2}(\partial_5 b), \quad \Gamma_{57}^4 = \frac{1}{2}(\partial_5 c), \\
&\Gamma_{58}^1 = \frac{1}{2}(\partial_8 a), \quad \Gamma_{58}^2 = \frac{1}{2}(\partial_8 b), \quad \Gamma_{58}^3 = \frac{1}{2}(\partial_5 c), \quad \Gamma_{66}^1 = \frac{1}{2}(2\partial_6 b), \quad \Gamma_{67}^1 = \frac{1}{2}(\partial_7 b), \\
&\Gamma_{67}^3 = \frac{1}{2}(\partial_6 b), \quad \Gamma_{67}^4 = \frac{1}{2}(\partial_6 c), \quad \Gamma_{68}^1 = \frac{1}{2}(\partial_8 b), \quad \Gamma_{68}^3 = \frac{1}{2}(\partial_6 c), \\
&\Gamma_{77}^1 = \frac{a}{2}(\partial_1 b) + \frac{b}{2}(\partial_2 b) + \frac{-1}{2}(\partial_5 b), \quad \Gamma_{77}^2 = \frac{b}{2}(\partial_1 b) + \frac{-1}{2}(\partial_6 b), \\
&\Gamma_{77}^3 = \frac{b}{2}(\partial_3 b) + \frac{c}{2}(\partial_4 b) + \frac{1}{2}(\partial_7 b), \quad \Gamma_{77}^4 = \frac{c}{2}(\partial_3 b) + \frac{1}{2}(2\partial_7 c - \partial_8 b), \\
&\Gamma_{77}^5 = \frac{-1}{2}(\partial_1 b), \quad \Gamma_{77}^6 = \frac{-1}{2}(\partial_2 b), \quad \Gamma_{77}^7 = \frac{-1}{2}(\partial_3 b), \quad \Gamma_{77}^8 = \frac{-1}{2}(\partial_4 b), \\
&\Gamma_{78}^1 = \frac{a}{2}(\partial_1 c) + \frac{b}{2}(\partial_2 c) + \frac{-1}{2}(\partial_5 c) \quad \Gamma_{78}^2 = \frac{b}{2}(\partial_1 c) + \frac{-1}{2}(\partial_6 c), \\
&\Gamma_{78}^3 = \frac{b}{2}(\partial_3 c) + \frac{c}{2}(\partial_4 c) + \frac{1}{2}(\partial_8 b), \quad \Gamma_{78}^4 = \frac{c}{2}(\partial_3 c), \quad \Gamma_{78}^5 = \frac{-1}{2}(\partial_1 c), \\
&\Gamma_{78}^6 = \frac{-1}{2}(\partial_2 c), \quad \Gamma_{78}^7 = \frac{-1}{2}(\partial_3 c), \quad \Gamma_{78}^8 = \frac{-1}{2}(\partial_4 c), \quad \Gamma_{88}^3 = \frac{1}{2}(2\partial_8 c).
\end{aligned}$$

We denote by ∇ the Levi Civita connection of a pseudo-Riemannian metric (1.1) and by R its curvature tensor, taken with the sign convention;

$$\nabla_{\partial_i} \partial_j = \sum_{k}^{8} \Gamma_{ij}^k \partial_k$$

$$R(X, Y) = [\nabla_X, \nabla_Y] - \nabla_{[X,Y]}. \tag{1.3}$$

From equations (1.3), after a long but straight forward calculations we obtained the following results:

Lemma 1.1. *The Non-zero components of the Levi-Civita connection of a Walker metric (1.1) are given by*

$$
\begin{aligned}
\nabla_{\partial_5}\partial_5 =& (\frac{a}{2}\partial_1 a + \frac{b}{2}\partial_2 a + \frac{1}{2}\partial_5 a)\partial_1 + (\frac{b}{2}\partial_1 a + \frac{1}{2}2\partial_5 b - \frac{1}{2}\partial_6 a)\partial_2 + (\frac{b}{2}\partial_3 a + \frac{c}{2}\partial_4 a \\
& + \frac{-1}{2}\partial_7 a)\partial_3 + (\frac{c}{2}\partial_3 a + \frac{-1}{2}\partial_8 a)\partial_4 + (\frac{-1}{2}\partial_1 a)\partial_5 + (\frac{-1}{2}\partial_2 a)\partial_6 + (\frac{-1}{2}\partial_3 a)\partial_7 \\
& + (\frac{-1}{2}\partial_4 a)\partial_8, \\
\nabla_{\partial_5}\partial_6 =& \frac{a}{2}\partial_1 b + \frac{b}{2}\partial_2 b + \frac{1}{2}\partial_6 a)\partial_1 + (\frac{b}{2}\partial_1 b)\partial_2 + (\frac{b}{2}\partial_3 b + \frac{c}{2}\partial_4 b + \frac{-1}{2}\partial_7 b)\partial_3 \\
& + (\frac{c}{2}\partial_3 b + \frac{-1}{2}\partial_8 b)\partial_4 + (\frac{-1}{2}\partial_1 b)\partial_5 + (\frac{-1}{2}\partial_2 b)\partial_6 + (\frac{-1}{2}\partial_3 b)\partial_7 + (\frac{-1}{2}\partial_4 b)\partial_8, \\
\nabla_{\partial_5}\partial_7 =& (\frac{1}{2}\partial_7 a)\partial_1 + (\frac{1}{2}\partial_7 b)\partial_2 + (\frac{1}{2}\partial_5 b)\partial_3 + (\frac{1}{2}\partial_5 c)\partial_4, \\
\nabla_{\partial_5}\partial_8 =& (\frac{1}{2}\partial_8 a)\partial_1 + (\frac{1}{2}\partial_8 b)\partial_2 + (\frac{1}{2}\partial_5 c)\partial_3, \quad \nabla_{\partial_6}\partial_6 = (\frac{1}{2}2\partial_6 b)\partial_1, \\
\nabla_{\partial_6}\partial_7 =& (\frac{1}{2}\partial_7 b)\partial_1 + (\frac{1}{2}\partial_6 b)\partial_3 + (\frac{1}{2}(\partial_6 c)\partial_4, \quad \nabla_{\partial_6}\partial_8 = (\frac{1}{2}\partial_8 b)\partial_1 + (\frac{1}{2}\partial_6 c)\partial_3, \\
\nabla_{\partial_7}\partial_7 =& (\frac{a}{2}\partial_1 b + \frac{b}{2}\partial_2 b + \frac{-1}{2}\partial_5 b)\partial_1 + (\frac{b}{2}\partial_1 b + \frac{-1}{2}\partial_6 b)\partial_2 + (\frac{b}{2}\partial_3 b + \frac{c}{2}\partial_4 b + \frac{1}{2}\partial_7 b)\partial_3 \\
& + (\frac{c}{2}\partial_3 b + \frac{1}{2}2\partial_7 c - \frac{1}{2}\partial_8 b)\partial_4 + (\frac{-1}{2}\partial_1 b)\partial_5 + (\frac{-1}{2}\partial_2 b)\partial_6 + (\frac{-1}{2}\partial_3 b)\partial_7 + (\frac{-1}{2}\partial_4 b)\partial_8, \\
\nabla_{\partial_7}\partial_8 =& (\frac{a}{2}\partial_1 c + \frac{b}{2}\partial_2 c + \frac{-1}{2}\partial_5 c)\partial_1 + (\frac{b}{2}\partial_1 c) + \frac{-1}{2}\partial_6 c)\partial_2 + (\frac{b}{2}\partial_3 c + \frac{c}{2}\partial_4 c + \frac{1}{2}\partial_8 b)\partial_3 \\
& + (\frac{c}{2}\partial_3 c)\partial_4 + (\frac{-1}{2}\partial_1 c)\partial_5 + (\frac{-1}{2}\partial_2 c)\partial_6 + (\frac{-1}{2}\partial_3 c)\partial_7 + (\frac{-1}{2}\partial_4 c)\partial_8, \\
\nabla_{\partial_8}\partial_8 =& \frac{1}{2}(2\partial_8 c)\partial_3.
\end{aligned}
$$

2 Non-strict Eight Dimensional Walker Manifold

In this section we define a Non-strict eight dimensional walker manifold as the one we can choose a coordinate system $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ such that the functions a, b and c are independend of the variables x_5, x_6, x_7 and x_8, that we mean $\frac{\partial a}{\partial x_i} = 0$, $\frac{\partial b}{\partial x_j} = 0$, and $\frac{\partial c}{\partial x_k} = 0$ for $i, j, k = 5, 6, 7, 8$. Here we let $\frac{\partial a}{\partial x_i} = a_i$, $\frac{\partial b}{\partial x_j} = b_j$, $\frac{\partial c}{\partial x_k} = c_k$, for $i, j, k = 1-4$ and let (1.1) be an eight dimensional Non-strict walker metric. We obtain the cononical form of the Levi-Civita connection, $(0, 4)$-curvature tensor, Ricci tensor, Weyl conformal curvature tensor and study some of the properties associated with the class of Non-strict $8 - dimensional$Walker manifold.

Proposition 2.1. *The Non-zero components of the Levi-Civita connections of the eight dimensional*

Non-strict walker metric (1.1) are as follows:

$$\begin{aligned}
\nabla_{\partial_5}\partial_5 =&(\frac{aa_1}{2}+\frac{ba_2}{2})\partial_1+\frac{ba_1}{2}\partial_2+(\frac{ba_3}{2}+\frac{ca_4}{2})\partial_3\\
&+(\frac{ca_3}{2})\partial_4-\frac{a_1}{2}\partial_5-\frac{a_2}{2}\partial_6-\frac{a_3}{2}\partial_7-\frac{a_4}{2}\partial_8,\\
\nabla_{\partial_5}\partial_6 =&(\frac{ab_1}{2}+\frac{bb_2}{2})\partial_1+(\frac{bb_1}{2})\partial_2+(\frac{bb_3}{2}+\frac{cb_4}{2})\partial_3\\
&+(\frac{cb_3}{2})\partial_4-(\frac{b_1}{2})\partial_5-(\frac{b_2}{2})\partial_6-(\frac{b_3}{2})\partial_7-(\frac{b_4}{2})\partial_8,\\
\nabla_{\partial_7}\partial_7 =&(\frac{ab_1}{2}+\frac{bb_2}{2})\partial_1+(\frac{bb_1}{2})\partial_2+(\frac{bb_3}{2}+\frac{cb_4}{2})\partial_3\\
&+(\frac{cb_3}{2})\partial_4-(\frac{b_1}{2})\partial_5-(\frac{b_2}{2})\partial_6-(\frac{b_3}{2})\partial_7-(\frac{b_4}{2})\partial_8,\\
\nabla_{\partial_7}\partial_8 =&(\frac{ac_1}{2}+\frac{bc_2}{2})\partial_1+(\frac{bc_1}{2})\partial_2+(\frac{bc_3}{2}+\frac{cc_4}{2})\partial_3\\
&+(\frac{cc_3}{2})\partial_4-(\frac{c_1}{2})\partial_5-(\frac{c_2}{2})\partial_6-(\frac{c_3}{2})\partial_7-(\frac{c_4}{2})\partial_8.
\end{aligned}$$

Proof. This is obtained from the Lemma (1.1). □

Using the proposition (2.1), the following is now immediate:

Lemma 2.1. *The Non-zero components of the curvature operator of the eight dimensional Non-strict walker metric (1.1) are given by;*

$$\begin{aligned}
&R(\partial_5,\partial_6)\partial_5=(\frac{a_1}{2}-\frac{b_2}{2})\nabla_{\partial_5}\partial_6-\frac{b_1}{2}\nabla_{\partial_5}\partial_5,\quad R(\partial_5,\partial_6)\partial_6=\frac{b_1}{2}\nabla_{\partial_6}\partial_5,\\
&R(\partial_5,\partial_7)\partial_5=\frac{a_3}{2}\nabla_{\partial_7}\partial_7+\frac{a_4}{2}\nabla_{\partial_7}\partial_8,\quad R(\partial_5,\partial_7)\partial_6=\frac{b_3}{2}\nabla_{\partial_7}\partial_7+\frac{b_4}{2}\nabla_{\partial_7}\partial_8,\\
&R(\partial_5,\partial_7)\partial_7=-\frac{b_1}{2}\nabla_{\partial_5}\partial_5-\frac{b_2}{2}\nabla_{\partial_5}\partial_6,\quad R(\partial_5,\partial_7)\partial_8=-\frac{c_1}{2}\nabla_{\partial_5}\partial_5-\frac{c_2}{2}\nabla_{\partial_5}\partial_6,\\
&R(\partial_5,\partial_8)\partial_5=\frac{a_3}{2}\nabla_{\partial_8}\partial_7,\quad R(\partial_5,\partial_8)\partial_6=\frac{b_3}{2}\nabla_{\partial_8}\partial_7,\quad R(\partial_5,\partial_8)\partial_7=-\frac{c_1}{2}\nabla_{\partial_5}\partial_5-\frac{c_2}{2}\nabla_{\partial_5}\partial_6,\\
&R(\partial_6,\partial_7)\partial_5=\frac{b_3}{2}\nabla_{\partial_7}\partial_7+\frac{b_4}{2}\nabla_{\partial_7}\partial_8,\quad R(\partial_6,\partial_7)\partial_7=-\frac{b_1}{2}\nabla_{\partial_6}\partial_5,\quad R(\partial_6,\partial_7)\partial_8=-\frac{c_1}{2}\nabla_{\partial_6}\partial_5,\\
&R(\partial_6,\partial_8)\partial_5=\frac{b_3}{2}\nabla_{\partial_8}\partial_7,\quad R(\partial_6,\partial_8)\partial_7=-\frac{c_1}{2}\nabla_{\partial_6}\partial_5,\\
&R(\partial_7,\partial_8)\partial_7=-\frac{c_3}{2}\nabla_{\partial_7}\partial_7-\frac{c_4}{2}\nabla_{\partial_7}\partial_8+\frac{b_3}{2}\nabla_{\partial_8}\partial_7,\quad R(\partial_7,\partial_8)\partial_8=\frac{c_3}{2}\nabla_{\partial_8}\partial_7
\end{aligned}$$

From Lemma (2.1), we can now determine all the $(0,4)$-curvature tensors $R(X,Y,Z,W)=g(R(X,Y)Z,W)$ with respect to ∂_i. From a long but routine calculations, we obtain the following results;

Theorem 2.2. *The nonzero components of the* $(0,4)$*-curvature tensor of the eight dimensional Non-strict walker metric (1.1) are given by:*

$$R(\partial_5,\partial_6,\partial_5,\partial_1)=\frac{b_1b_2}{4}-\frac{a_1b_1}{4}+\frac{a_1b_1}{4}=\frac{b_1b_2}{4},\quad R(\partial_5,\partial_6,\partial_5,\partial_2)=\frac{b_2b_2}{4}-\frac{a_1b_2}{4}+\frac{a_2b_1}{4},$$
$$R(\partial_5,\partial_6,\partial_5,\partial_3)=\frac{b_2b_3}{4}-\frac{a_1b_3}{4}+\frac{a_3b_1}{4},\quad R(\partial_5,\partial_6,\partial_5,\partial_4)=\frac{b_2b_4}{4}-\frac{a_1b_4}{4}+\frac{a_4b_1}{4},$$
$$R(\partial_5,\partial_6,\partial_6,\partial_1)=-\frac{b_1^2}{4},\quad R(\partial_5,\partial_6,\partial_6,\partial_2)=-\frac{b_1b_2}{4},$$
$$R(\partial_5,\partial_6,\partial_6,\partial_3)=-\frac{b_1b_3}{4},\quad R(\partial_5,\partial_6,\partial_6,\partial_4)=-\frac{b_1b_4}{4},$$
$$R(\partial_5,\partial_7,\partial_5,\partial_1)=-\frac{a_3b_1}{4}-\frac{a_4c_1}{4},\quad R(\partial_5,\partial_7,\partial_5,\partial_2)=-\frac{a_3b_2}{4}-\frac{a_4c_2}{4},$$
$$R(\partial_5,\partial_7,\partial_5,\partial_3)=-\frac{a_3b_3}{4}-\frac{a_4c_3}{4},\quad R(\partial_5,\partial_7,\partial_5,\partial_4)=-\frac{a_3b_4}{4}-\frac{a_4c_4}{4},$$
$$R(\partial_5,\partial_7,\partial_6,\partial_1)=-\frac{b_3b_1}{4}-\frac{b_4c_1}{4},\quad R(\partial_5,\partial_7,\partial_6,\partial_2)=-\frac{b_3b_2}{4}-\frac{b_4c_2}{4},$$
$$R(\partial_5,\partial_7,\partial_6,\partial_3)=-\frac{b_3b_3}{4}-\frac{b_4c_3}{4},\quad R(\partial_5,\partial_7,\partial_6,\partial_4)=-\frac{b_3b_4}{4}-\frac{b_4c_4}{4},$$
$$R(\partial_5,\partial_7,\partial_7,\partial_1)=\frac{b_1a_1}{4}+\frac{b_2b_1}{4},\quad R(\partial_5,\partial_7,\partial_7,\partial_2)=\frac{b_1a_2}{4}+\frac{b_2b_2}{4},$$
$$R(\partial_5,\partial_7,\partial_7,\partial_3)=\frac{b_1a_3}{4}+\frac{b_2b_3}{4},\quad R(\partial_5,\partial_7,\partial_7,\partial_4)=\frac{b_1a_4}{4}+\frac{b_2b_4}{4},$$
$$R(\partial_5,\partial_7,\partial_8,\partial_1)=\frac{c_1a_1}{4}+\frac{c_2b_1}{4},\quad R(\partial_5,\partial_7,\partial_8,\partial_2)=\frac{c_1a_2}{4}+\frac{c_2b_2}{4},$$
$$R(\partial_5,\partial_7,\partial_8,\partial_3)=\frac{c_1a_3}{4}+\frac{c_2b_3}{4},\quad R(\partial_5,\partial_7,\partial_8,\partial_4)=\frac{c_1a_4}{4}+\frac{c_2b_4}{4},$$
$$R(\partial_5,\partial_8,\partial_5,\partial_1)=-\frac{a_3c_1}{4},\quad R(\partial_5,\partial_8,\partial_5,\partial_2)=-\frac{a_3c_2}{4},$$
$$R(\partial_5,\partial_8,\partial_5,\partial_3)=-\frac{a_3c_3}{4},\quad R(\partial_5,\partial_8,\partial_5,\partial_4)=-\frac{a_3c_4}{4},$$
$$R(\partial_5,\partial_8,\partial_6,\partial_1)=-\frac{b_3c_1}{4},\quad R(\partial_5,\partial_8,\partial_6,\partial_2)=-\frac{b_3c_2}{4},$$
$$R(\partial_5,\partial_8,\partial_6,\partial_3)=-\frac{b_3c_3}{4},\quad R(\partial_5,\partial_8,\partial_6,\partial_4)=-\frac{b_3c_4}{4},$$
$$R(\partial_5,\partial_8,\partial_7,\partial_1)=\frac{c_1a_1}{4}+\frac{c_2b_1}{4},\quad R(\partial_5,\partial_8,\partial_7,\partial_2)=\frac{c_1a_2}{4}+\frac{c_2b_2}{4},$$
$$R(\partial_5,\partial_8,\partial_7,\partial_3)=\frac{c_1a_3}{4}+\frac{c_2b_3}{4},\quad R(\partial_5,\partial_8,\partial_7,\partial_4)=\frac{c_1a_4}{4}+\frac{c_2b_4}{4},$$
$$R(\partial_6,\partial_7,\partial_5,\partial_1)=-\frac{b_3b_1}{4}-\frac{b_4c_1}{4},\quad R(\partial_6,\partial_7,\partial_5,\partial_2)=-\frac{b_3b_2}{4}-\frac{b_4c_2}{4},$$
$$R(\partial_6,\partial_7,\partial_5,\partial_3)=-\frac{b_3b_3}{4}-\frac{b_4c_3}{4},\quad R(\partial_6,\partial_7,\partial_5,\partial_4)=-\frac{b_3b_4}{4}-\frac{b_4c_4}{4},$$
$$R(\partial_6,\partial_7,\partial_7,\partial_1)=\frac{b_1b_1}{4},\quad R(\partial_6,\partial_7,\partial_7,\partial_2)=\frac{b_1b_2}{4},$$
$$R(\partial_6,\partial_7,\partial_7,\partial_3)=\frac{b_1b_3}{4},\quad R(\partial_6,\partial_7,\partial_7,\partial_4)=\frac{b_1b_4}{4},$$
$$R(\partial_6,\partial_7,\partial_8,\partial_1)=\frac{c_1b_1}{4},\quad R(\partial_6,\partial_7,\partial_8,\partial_2)=\frac{c_1b_2}{4},$$
$$R(\partial_6,\partial_7,\partial_8,\partial_3)=\frac{c_1b_3}{4},\quad R(\partial_6,\partial_7,\partial_8,\partial_4)=\frac{c_1b_4}{4},$$
$$R(\partial_6,\partial_8,\partial_5,\partial_1)=-\frac{b_3c_1}{4},\quad R(\partial_6,\partial_8,\partial_5,\partial_2)=-\frac{b_3c_2}{4},$$
$$R(\partial_6,\partial_8,\partial_5,\partial_3)=-\frac{b_3c_3}{4},\quad R(\partial_6,\partial_8,\partial_5,\partial_4)=-\frac{b_3c_4}{4},$$
$$R(\partial_6,\partial_8,\partial_7,\partial_1)=\frac{c_1b_1}{4},\quad R(\partial_6,\partial_8,\partial_7,\partial_2)=\frac{c_1b_2}{4}$$

$$R(\partial_6,\partial_8,\partial_7,\partial_3)=\frac{c_1b_3}{4},\quad R(\partial_6,\partial_8,\partial_7,\partial_4)=\frac{c_1b_4}{4},$$
$$R(\partial_7,\partial_8,\partial_7,\partial_1)=\frac{c_3b_1}{4}+\frac{c_4c_1}{4}-\frac{b_3c_1}{4},\quad R(\partial_7,\partial_8,\partial_7,\partial_2)=\frac{c_3b_2}{4}+\frac{c_4c_2}{4}-\frac{b_3c_2}{4},$$
$$R(\partial_7,\partial_8,\partial_7,\partial_3)=\frac{c_3b_3}{4}+\frac{c_4c_3}{4}-\frac{b_3c_3}{4},\quad R(\partial_7,\partial_8,\partial_7,\partial_4)=\frac{c_3b_4}{4}+\frac{c_4c_4}{4}-\frac{b_3c_4}{4},$$
$$R(\partial_7,\partial_8,\partial_8,\partial_1)=-\frac{c_3c_1}{4},\quad R(\partial_7,\partial_8,\partial_8,\partial_2)=-\frac{c_3c_2}{4},$$
$$R(\partial_7,\partial_8,\partial_8,\partial_3)=-\frac{c_3c_3}{4},\quad R(\partial_7,\partial_8,\partial_8,\partial_4)=-\frac{c_3c_4}{4}.$$

The Ricci tensor is defined as $Ric(x,y)=trace\{z\to R(X,Z)Y\}$ and so from theorem (2.2) using the equation;

$$Ricc(X,Y)=\sum_{i,j=1}^{8}g^{ij}R(X,\partial_i,\partial_j,Y),$$

we have the following results:

Theorem 2.3. *The Non-zero components of the Ricci tensor of the eight dimensional Non-strict walker metric (1.1) are:*

$$Ricc(\partial_5,\partial_5)=\frac{a_1b_2}{2}-\frac{a_2b_1}{2}-\frac{b_2^2}{2}+\frac{a_3b_3}{2}+\frac{a_4c_3}{2}+\frac{a_3c_4}{2},$$
$$Ricc(\partial_5,\partial_6)=\frac{b_2b_1}{2}+\frac{b_3^2}{2}+\frac{b_4c_3}{2}+\frac{b_3c_4}{2},\quad Ricc(\partial_5,\partial_7)=-(\frac{a_3b_1}{2}+\frac{a_4c_1}{2}+\frac{b_2b_3}{2}+\frac{b_4c_2}{2}),$$
$$Ricc(\partial_5,\partial_8)=-(\frac{a_3c_1}{2}+\frac{b_3c_2}{2}),\quad Ricc(\partial_6,\partial_6)=-\frac{b_1^2}{2},\quad Ricc(\partial_6,\partial_7)=-\frac{b_1b_3}{2}-\frac{c_1b_4}{2},$$
$$Ricc(\partial_6,\partial_8)=-\frac{c_1b_3}{2},\quad Ricc(\partial_7,\partial_7)=\frac{b_2b_1}{2}+\frac{b_2b_1}{2}+\frac{b_1a_1}{2}-(\frac{c_3b_4}{2}+\frac{c_4^2}{2}-\frac{b_3c_4}{2}),$$
$$Ricc(\partial_7,\partial_8)=\frac{a_1c_1}{2}+\frac{b_1c_2}{2}+\frac{b_2c_1}{2}+\frac{c_3c_4}{2},\quad Ricc(\partial_8,\partial_8)=-\frac{c_3^2}{2}$$

Let τ denote the scalar curvature of the Non-strict walker metric (1.1). We define the scalar curvature by the equation

$$\tau=\sum_{i,j=1}^{8}g^{ij}Ricc(i,j)$$

Thus, we have the following result;

Theorem 2.4. *The eight dimensional Non-strict walker metric (1.1) has zero scalar curvature.*

Proof. Observe that the metric (1.1) is of the form

$$g_{ij}=\begin{pmatrix}0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\\1&0&0&0&a&b&0&0\\0&1&0&0&b&0&0&0\\0&0&1&0&0&0&b&c\\0&0&0&1&0&0&c&0\end{pmatrix}\text{ and } g^{ij}=\begin{pmatrix}-a&-b&0&0&1&0&0&0\\-b&0&0&0&0&1&0&0\\0&0&-b&-c&0&0&1&0\\0&0&-c&0&0&0&0&1\\1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\end{pmatrix},$$

where g^{ij} is the inverse of the metric g_{ij}. From the equation $\tau=\sum_{i,j=1}^{8}g^{ij}Ricc(i,j)$ and theorem (2.3) we observe that $Ricc(ij)=0$ for $i,j=1,2,3,4$ and $g^{ij}=0$ for $i,j=5,6,7,8$. Thus $\tau=0$ for all $i,j=1,2,...,8$. $\square$

Definition 2.1. A Walker metric is said to be Einstein Walker metric if its Ricci tensor is a scalar multiple of the metric at each point.

This definition implies that the eight dimensional nonstrict walker metric (1.1) is an Einstein Walker metric if there is a constant μ so that $Ricc = \mu g$. The Schouten tensor C_{ij} is define by the equation $C_{ij} = Ricc(ij) - \frac{\tau}{(2m-1)} g_{ij}$. Thus we have the follwing;

Theorem 2.5. *The eight dimensional Non-strict walker metric (1.1) is not Einstein.*

Proof. Since Schouten equation is given by $C_{ij} = Ric(i,j) - \frac{\tau}{(2m-1)} g_{ij}$ and from theorem (2.4) the scalar curvature $\tau = 0$. Therefore, $C_{ij} = Ric(i,j)$ and the result follows. $\square$

The Weyl conformal curvature tensor $\mathfrak{W}$ is defined by the equation

$$\mathfrak{W}(X,Y,Z,W) = R(X,Y,Z,W) + \frac{\tau}{(m-1)(m-2)}\{g(X,W)g(Y,Z) - g(X,Z)g(Y,W)\}$$
$$+ \frac{1}{(n-2)}\{Ricc(Y,Z)g(X,W) - Ricc(X,Z)g(Y,W) - Ricc(Y,W)g(X,Z)$$
$$- Ricc(X,W)g(Y,Z)\}. \tag{2.1}$$

Since the Scalar curvature $\tau = 0$, for the eight dimensional Non-strict metric (1.1), the Weyl conformal curvature tensor $\mathfrak{W}$ becomes

$$\begin{aligned}\mathfrak{W}(X,Y,Z,W) =& R(X,Y,Z,W) + \frac{1}{n-2}\{\rho(Y,Z)g(X,W) - \rho(X,Z)g(Y,W) \\ &- \rho(Y,W)g(X,Z) + \rho(X,W)g(Y,Z)\}.\end{aligned}$$

Lemma 2.6. *The Non-zero components of the Weyl conformal tensor of the eight dimensional Non-strict walker metric (1.1) is given by;*

$$\mathfrak{W}(\partial_5,\partial_6,\partial_5,\partial_1)=\frac{b_1b_2}{4}+\frac{1}{6}\{\frac{b_2b_1}{2}+\frac{b_3^2}{2}+\frac{b_4c_3}{2}+\frac{b_3c_4}{2}\},$$
$$\mathfrak{W}(\partial_5,\partial_6,\partial_5,\partial_2)=\frac{b_2b_2}{4}-\frac{a_1b_2}{4}+\frac{a_2b_1}{4}+\frac{1}{6}\{-(\frac{a_1b_2}{2}-\frac{a_2b_1}{2}-\frac{b_2^2}{2}+\frac{a_3b_3}{2}+\frac{a_4c_3}{2}+\frac{a_3c_4}{2})\},$$
$$\mathfrak{W}(\partial_5,\partial_6,\partial_5,\partial_3)=\frac{b_2b_3}{4}-\frac{a_1b_3}{4}+\frac{a_3b_1}{4},\quad \mathfrak{W}(\partial_5,\partial_6,\partial_5,\partial_4)=\frac{b_2b_4}{4}-\frac{a_1b_4}{4}+\frac{a_4b_1}{4},$$
$$\mathfrak{W}(\partial_5,\partial_6,\partial_6,\partial_1)=-\frac{b_1^2}{4}+\frac{1}{6}\{-\frac{b_1^2}{2}\},\quad \mathfrak{W}(\partial_5,\partial_6,\partial_6,\partial_2)=-\frac{b_1b_2}{4}+\frac{1}{6}\{-(\frac{b_2b_1}{2}+\frac{b_3^2}{2}+\frac{b_4c_3}{2}+\frac{b_3c_4}{2})\},$$
$$\mathfrak{W}(\partial_5,\partial_6,\partial_6,\partial_3)=-\frac{b_1b_3}{4},\quad \mathfrak{W}(\partial_5,\partial_6,\partial_6,\partial_4)=-\frac{b_1b_4}{4},$$
$$\mathfrak{W}(\partial_5,\partial_7,\partial_5,\partial_1)=-\frac{a_3b_1}{4}-\frac{a_4c_1}{4}+\frac{1}{6}\{-(\frac{a_3b_1}{2}+\frac{a_4c_1}{2}+\frac{b_2b_3}{2}+\frac{b_4c_2}{2})\},$$
$$\mathfrak{W}(\partial_5,\partial_7,\partial_5,\partial_2)=-\frac{a_3b_2}{4}-\frac{a_4c_2}{4},$$
$$\mathfrak{W}(\partial_5,\partial_7,\partial_5,\partial_3)=-\frac{a_3b_3}{4}-\frac{a_4c_3}{4}+\frac{1}{6}\{-(\frac{a_1b_2}{2}-\frac{a_2b_1}{2}-\frac{b_2^2}{2}+\frac{a_3b_3}{2}+\frac{a_4c_3}{2}+\frac{a_3c_4}{2})\},$$
$$\mathfrak{W}(\partial_5,\partial_7,\partial_5,\partial_4)=-\frac{a_3b_4}{4}-\frac{a_4c_4}{4},\quad \mathfrak{W}(\partial_5,\partial_7,\partial_6,\partial_1)=-\frac{b_3b_1}{4}-\frac{b_4c_1}{4}+\frac{1}{6}\{-\frac{b_1b_3}{2}-\frac{c_1b_4}{2}\},$$
$$\mathfrak{W}(\partial_5,\partial_7,\partial_6,\partial_2)=-\frac{b_3b_2}{4}-\frac{b_4c_2}{4},\quad \mathfrak{W}(\partial_5,\partial_7,\partial_6,\partial_3)=-\frac{b_3b_3}{4}-\frac{b_4c_3}{4}+\frac{1}{6}\{-(\frac{b_2b_1}{2}+\frac{b_3^2}{2}+\frac{b_4c_3}{2}+\frac{b_3c_4}{2})\},$$
$$\mathfrak{W}(\partial_5,\partial_7,\partial_6,\partial_4)=-\frac{b_3b_4}{4}-\frac{b_4c_4}{4},$$
$$\mathfrak{W}(\partial_5,\partial_7,\partial_7,\partial_1)=\frac{b_1a_1}{4}+\frac{b_2b_1}{4}+\frac{1}{6}\{\frac{b_2b_1}{2}+\frac{b_2b_1}{2}+\frac{b_1a_1}{2}-(\frac{c_3b_4}{2}+\frac{c_4^2}{2}-\frac{b_3c_4}{2})\},$$
$$\mathfrak{W}(\partial_5,\partial_7,\partial_7,\partial_2)=\frac{b_1a_2}{4}+\frac{b_2b_2}{4},\quad \mathfrak{W}(\partial_5,\partial_7,\partial_7,\partial_3)=\frac{b_1a_3}{4}+\frac{b_2b_3}{4}+\frac{1}{6}\{(\frac{a_3b_1}{2}+\frac{a_4c_1}{2}+\frac{b_2b_3}{2}+\frac{b_4c_2}{2})\},$$
$$\mathfrak{W}(\partial_5,\partial_7,\partial_7,\partial_4)=\frac{b_1a_4}{4}+\frac{b_2b_4}{4},\quad \mathfrak{W}(\partial_5,\partial_7,\partial_8,\partial_1)=\frac{c_1a_1}{4}+\frac{c_2b_1}{4}+\frac{1}{6}\{\frac{a_1c_1}{2}+\frac{b_1c_2}{2}+\frac{b_2c_1}{2}+\frac{c_3c_4}{2}\},$$
$$\mathfrak{W}(\partial_5,\partial_7,\partial_8,\partial_2)=\frac{c_1a_2}{4}+\frac{c_2b_2}{4},\quad \mathfrak{W}(\partial_5,\partial_7,\partial_8,\partial_3)=\frac{c_1a_3}{4}+\frac{c_2b_3}{4}+\frac{1}{6}\{\frac{a_3c_1}{2}+\frac{b_3c_2}{2}\},$$
$$\mathfrak{W}(\partial_5,\partial_7,\partial_8,\partial_4)=\frac{c_1a_4}{4}+\frac{c_2b_4}{4},$$

$$\mathfrak{W}(\partial_5,\partial_8,\partial_5,\partial_1)=-\frac{a_3c_1}{4}+\frac{1}{6}\{-(\frac{a_3c_1}{2}+\frac{b_3c_2}{2})\},\quad \mathfrak{W}(\partial_5,\partial_8,\partial_5,\partial_2)=-\frac{a_3c_2}{4},$$
$$\mathfrak{W}(\partial_5,\partial_8,\partial_5,\partial_3)=-\frac{a_3c_3}{4},$$
$$\mathfrak{W}(\partial_5,\partial_8,\partial_5,\partial_4)=-\frac{a_3c_4}{4}+\frac{1}{6}\{-(\frac{a_1b_2}{2}-\frac{a_2b_1}{2}-\frac{b_2^2}{2}+\frac{a_3b_3}{2}+\frac{a_4c_3}{2}+\frac{a_3c_4}{2})\},$$
$$\mathfrak{W}(\partial_5,\partial_8,\partial_6,\partial_1)=-\frac{b_3c_1}{4}+\frac{1}{6}\{-\frac{c_1b_3}{2}\},\quad \mathfrak{W}(\partial_5,\partial_8,\partial_6,\partial_2)=-\frac{b_3c_2}{4},\quad \mathfrak{W}(\partial_5,\partial_8,\partial_6,\partial_3)=-\frac{b_3c_3}{4},$$
$$\mathfrak{W}(\partial_5,\partial_8,\partial_6,\partial_4)=-\frac{b_3c_4}{4}+\frac{1}{6}\{-(\frac{b_2b_1}{2}+\frac{b_3^2}{2}+\frac{b_4c_3}{2}+\frac{b_3c_4}{2})\},$$

$$\mathfrak{W}(\partial_5,\partial_8,\partial_7,\partial_1)=\frac{c_1a_1}{4}+\frac{c_2b_1}{4}+\frac{1}{6}\{\frac{a_1c_1}{2}+\frac{b_1c_2}{2}+\frac{b_2c_1}{2}+\frac{c_3c_4}{2}\},$$
$$\mathfrak{W}(\partial_5,\partial_8,\partial_7,\partial_2)=\frac{c_1a_2}{4}+\frac{c_2b_2}{4},\quad \mathfrak{W}(\partial_5,\partial_8,\partial_7,\partial_3)=\frac{c_1a_3}{4}+\frac{c_2b_3}{4},$$
$$\mathfrak{W}(\partial_5,\partial_8,\partial_7,\partial_4)=\frac{c_1a_4}{4}+\frac{c_2b_4}{4}+\frac{1}{6}\{(\frac{a_3b_1}{2}+\frac{a_4c_1}{2}+\frac{b_2b_3}{2}+\frac{b_4c_2}{2})\},$$
$$\mathfrak{W}(\partial_6,\partial_7,\partial_5,\partial_1)=-\frac{b_3b_1}{4}-\frac{b_4c_1}{4},$$
$$\mathfrak{W}(\partial_6,\partial_7,\partial_5,\partial_2)=-\frac{b_3b_2}{4}-\frac{b_4c_2}{4}+\frac{1}{6}\{-(\frac{a_3b_1}{2}+\frac{a_4c_1}{2}+\frac{b_2b_3}{2}+\frac{b_4c_2}{2})\},$$
$$\mathfrak{W}(\partial_6,\partial_7,\partial_5,\partial_3)=-\frac{b_3b_3}{4}-\frac{b_4c_3}{4}+\frac{1}{6}\{-(\frac{b_2b_1}{2}+\frac{b_3^2}{2}+\frac{b_4c_3}{2}+\frac{b_3c_4}{2})\},$$
$$\mathfrak{W}(\partial_6,\partial_7,\partial_5,\partial_4)=-\frac{b_3b_4}{4}-\frac{b_4c_4}{4},\quad \mathfrak{W}(\partial_6,\partial_7,\partial_7,\partial_1)=\frac{b_1b_1}{4},$$
$$\mathfrak{W}(\partial_6,\partial_7,\partial_7,\partial_2)=\frac{b_1b_2}{4}+\frac{1}{6}\{\frac{b_2b_1}{2}+\frac{b_2b_1}{2}+\frac{b_1a_1}{2}-(\frac{c_3b_4}{2}+\frac{c_4^2}{2}-\frac{b_3c_4}{2})\},$$
$$\mathfrak{W}(\partial_6,\partial_7,\partial_7,\partial_3)=\frac{b_1b_3}{4}+\frac{1}{6}\{-(\frac{b_1b_3}{2}+\frac{c_1b_4}{2})\},\quad \mathfrak{W}(\partial_6,\partial_7,\partial_7,\partial_4)=\frac{b_1b_4}{4},$$
$$\mathfrak{W}(\partial_6,\partial_7,\partial_8,\partial_1)=\frac{c_1b_1}{4},\quad \mathfrak{W}(\partial_6,\partial_7,\partial_8,\partial_2)=\frac{c_1b_2}{4}+\frac{1}{6}\{\frac{a_1c_1}{2}+\frac{b_1c_2}{2}+\frac{b_2c_1}{2}+\frac{c_3c_4}{2}\},$$
$$\mathfrak{W}(\partial_6,\partial_7,\partial_8,\partial_3)=\frac{c_1b_3}{4}+\frac{1}{6}\{\frac{c_1b_3}{2}\},\quad \mathfrak{W}(\partial_6,\partial_7,\partial_8,\partial_4)=\frac{c_1b_4}{4},$$

$$\mathfrak{W}(\partial_6,\partial_8,\partial_5,\partial_1)=-\frac{b_3c_1}{4},\quad \mathfrak{W}(\partial_6,\partial_8,\partial_5,\partial_2)=-\frac{b_3c_2}{4}+\frac{1}{6}\{-(\frac{a_3c_1}{2}+\frac{b_3c_2}{2})\},$$
$$\mathfrak{W}(\partial_6,\partial_8,\partial_5,\partial_3)=-\frac{b_3c_3}{4},\quad \mathfrak{W}(\partial_6,\partial_8,\partial_5,\partial_4)=-\frac{b_3c_4}{4}+\frac{1}{6}\{-\frac{b_2b_1}{2}+\frac{b_3^2}{2}+\frac{b_4c_3}{2}+\frac{b_3c_4}{2}\},$$
$$\mathfrak{W}(\partial_6,\partial_8,\partial_7,\partial_1)=\frac{c_1b_1}{4},\quad \mathfrak{W}(\partial_6,\partial_8,\partial_7,\partial_2)=\frac{c_1b_2}{4}+\frac{1}{6}\{\frac{a_1c_1}{2}+\frac{b_1c_2}{2}+\frac{b_2c_1}{2}+\frac{c_3c_4}{2}\},$$
$$\mathfrak{W}(\partial_6,\partial_8,\partial_7,\partial_3)=\frac{c_1b_3}{4},\quad \mathfrak{W}(\partial_6,\partial_8,\partial_7,\partial_4)=\frac{c_1b_4}{4}+\frac{1}{6}\{-(\frac{b_1b_3}{2}+\frac{c_1b_4}{2})\},$$
$$\mathfrak{W}(\partial_7,\partial_8,\partial_7,\partial_1)=\frac{c_3b_1}{4}+\frac{c_4c_1}{4}-\frac{b_3c_1}{4},\quad \mathfrak{W}(\partial_7,\partial_8,\partial_7,\partial_2)=\frac{c_3b_2}{4}+\frac{c_4c_2}{4}-\frac{b_3c_2}{4},$$
$$\mathfrak{W}(\partial_7,\partial_8,\partial_7,\partial_3)=\frac{c_3b_3}{4}+\frac{c_4c_3}{4}-\frac{b_3c_3}{4}+\frac{1}{6}\{\frac{a_1c_1}{2}+\frac{b_1c_2}{2}+\frac{b_2c_1}{2}+\frac{c_3c_4}{2}\},$$
$$\mathfrak{W}(\partial_7,\partial_8,\partial_7,\partial_4)=\frac{c_3b_4}{4}+\frac{c_4c_4}{4}-\frac{b_3c_4}{4}+\frac{1}{6}\{-\frac{b_2b_1}{2}-\frac{b_2b_1}{2}-\frac{b_1a_1}{2}+\frac{c_3b_4}{2}+\frac{c_4^2}{2}-\frac{b_3c_4}{2}\},$$
$$\mathfrak{W}(\partial_7,\partial_8,\partial_8,\partial_1)=-\frac{c_3c_1}{4},\quad \mathfrak{W}(\partial_7,\partial_8,\partial_8,\partial_2)=-\frac{c_3c_2}{4},$$
$$\mathfrak{W}(\partial_7,\partial_8,\partial_8,\partial_3)=-\frac{c_3c_3}{4}+\frac{1}{6}\{-\frac{c_3^2}{2}\},\quad \mathfrak{W}(\partial_7,\partial_8,\partial_8,\partial_4)=-\frac{c_3c_4}{4}.$$

Definition 2.2. A pseudo-Riemannian manifold is locally conformally flat if and only if its Weyl tensor vanishes.

Theorem 2.7. *The eight dimensional Non-strict walker metric (1.1) is locally conformally flat if and only if the functions a, b and c are constants or they satisfy the partial differential equation*

$$\frac{b_1b_2}{3}+\frac{b_3^2}{12}+\frac{b_4c_3}{12}+\frac{b_3c_4}{12}=0, \quad \frac{b_2^2}{3}-\frac{a_1b_2}{3}+\frac{a_2b_1}{3}-\frac{a_3b_3}{2}+\frac{a_4c_3}{2}+\frac{a_3c_4}{2}=0,$$
$$\frac{b_2b_3}{4}-\frac{a_1b_3}{4}+\frac{a_3b_1}{4}=0, \quad \frac{b_2b_4}{4}-\frac{a_1b_4}{4}+\frac{a_4b_1}{4}=0, \quad -\frac{b_1^2}{3}=0,$$
$$-\frac{b_1b_2}{3}-\frac{b_3^2}{12}-\frac{b_4c_3}{12}-\frac{b_3c_4}{12}=0, \quad -\frac{b_1b_3}{4}=0, \quad -\frac{b_1b_4}{4}=0, \quad -\frac{a_3b_1}{3}-\frac{a_4c_1}{3}-\frac{b_2b_3}{2}-\frac{b_4c_2}{2}=0,$$
$$-\frac{a_3b_2}{4}-\frac{a_4c_2}{4}=0, \quad -\frac{a_3b_3}{3}-\frac{a_4c_3}{3}-\frac{a_1b_2}{12}+\frac{a_2b_1}{12}+\frac{b_2^2}{12}-\frac{a_3c_4}{12}=0, \quad -\frac{a_3b_4}{4}-\frac{a_4c_4}{4}=0,$$
$$-\frac{b_3b_1}{4}-\frac{b_4c_1}{4}=0, \quad -\frac{b_3b_2}{4}-\frac{b_4c_2}{4}=0, \quad -\frac{b_3^2}{3}-\frac{b_4c_3}{3}-\frac{b_2b_1}{2}-\frac{b_3c_4}{2}=0, \quad -\frac{b_3b_4}{4}-\frac{b_4c_4}{4}=0,$$
$$\frac{b_1a_1}{3}+\frac{5b_2b_1}{12}-\frac{c_3b_4}{2}-\frac{c_4^2}{2}+\frac{b_3c_4}{2}=0, \quad \frac{b_1a_2}{4}+\frac{b_2b_2}{4}=0, \quad \frac{b_1a_3}{3}+\frac{b_2b_3}{3}+\frac{a_4c_1}{12}+\frac{b_4c_2}{12}=0,$$
$$\frac{b_1a_4}{4}+\frac{b_2b_4}{4}=0, \quad \frac{c_1a_1}{3}+\frac{c_2b_1}{3}+\frac{b_2c_1}{12}+\frac{c_3c_4}{12}=0, \quad \frac{c_1a_2}{4}+\frac{c_2b_2}{4}=0, \quad \frac{c_1a_3}{3}+\frac{c_2b_3}{3}=0,$$
$$\frac{c_1a_4}{4}+\frac{c_2b_4}{4}=0, \quad -\frac{a_3c_1}{3}-\frac{b_3c_2}{2}=0, \quad -\frac{a_3c_2}{4}=0, \quad -\frac{a_3c_3}{4}=0,$$
$$-\frac{a_3c_4}{3}-\frac{a_1b_2}{12}+\frac{a_2b_1}{12}+\frac{b_2^2}{12}-\frac{a_3b_3}{12}-\frac{a_4c_3}{12}=0,$$

$$-\frac{b_3c_1}{3}=0, \quad -\frac{b_3c_2}{4}=0, \quad -\frac{b_3c_3}{4}=0, \quad -\frac{b_3c_4}{3}-\frac{b_2b_1}{2}-\frac{b_3^2}{2}-\frac{b_4c_3}{2}=0,$$
$$\frac{c_1a_1}{3}+\frac{c_2b_1}{3}+\frac{b_2c_1}{12}+\frac{c_3c_4}{12}=0, \quad \frac{c_1a_2}{4}+\frac{c_2b_2}{4}=0, \quad \frac{c_1a_3}{4}+\frac{c_2b_3}{4}=0,$$
$$\frac{c_1a_4}{3}+\frac{c_2b_4}{3}+\frac{a_3b_1}{12}+\frac{b_2b_3}{12}=0, \quad -\frac{b_3b_1}{4}-\frac{b_4c_1}{4}=0, \quad -\frac{b_3b_2}{3}-\frac{b_4c_2}{3}-\frac{a_3b_1}{12}-\frac{a_4c_1}{12}=0,$$
$$-\frac{b_3^2}{3}-\frac{b_4c_3}{3}-\frac{b_2b_1}{12}-\frac{b_3c_4}{12}\}=0, \quad -\frac{b_3b_4}{4}-\frac{b_4c_4}{4}=0, \quad =\frac{b_1b_1}{4}=0,$$
$$\frac{5b_1b_2}{12}+\frac{b_1a_1}{12}-\frac{c_3b_4}{12}-\frac{c_4^2}{12}+\frac{b_3c_4}{12}=0, \quad \frac{b_1b_3}{3}-\frac{c_1b_4}{12})=0, \quad \frac{b_1b_4}{4}=0, \quad \frac{c_1b_1}{4}=0,$$
$$\frac{c_1b_2}{3}+\frac{a_1c_1}{12}+\frac{b_1c_2}{12}+\frac{c_3c_4}{12}=0, \quad \frac{c_1b_3}{3}=0, \quad \frac{c_1b_4}{4}=0, \quad -\frac{b_3c_1}{4}=0, \quad -\frac{b_3c_2}{3}-\frac{a_3c_1}{2}=0,$$
$$-\frac{b_3c_3}{4}=0, \quad -\frac{b_3c_4}{3}-\frac{b_2b_1}{12}-\frac{b_3^2}{12}-\frac{b_4c_3}{12}=0, \quad \frac{c_1b_1}{4}=0, \quad \frac{c_1b_2}{3}+\frac{a_1c_1}{12}+\frac{b_1c_2}{12}+\frac{c_3c_4}{12}=0,$$
$$\frac{c_1b_3}{4}=0, \quad \frac{c_1b_4}{3}-(\frac{b_1b_3}{12}=0, \quad \frac{c_3b_1}{4}+\frac{c_4c_1}{4}-\frac{b_3c_1}{4}=0, \quad \frac{c_3b_2}{4}+\frac{c_4c_2}{4}-\frac{b_3c_2}{4}=0,$$
$$\frac{c_4c_3}{3}+\frac{a_1c_1}{12}+\frac{b_1c_2}{12}+\frac{b_2c_1}{12}=0, \quad \frac{c_3b_4}{3}+\frac{c_4^2}{3}-\frac{b_3c_4}{3}-\frac{b_2b_1}{12}-\frac{b_2b_1}{12}-\frac{b_1a_1}{12}=0,$$
$$-\frac{c_3c_1}{4}=0, \quad -\frac{c_3c_2}{4}=0, \quad -\frac{c_3^2}{3}\}=0, \quad -\frac{c_3c_4}{4}=0.$$

Proof. From Lemma (2.6), if a, b and c are constans, then $\mathfrak{W}(\partial_i, \partial_j, \partial_k, \partial_l) = 0$ for all $i, j, k = 5, 6, 7, 8$ and $l = 1, 2, 3, 4$ $\square$

3 Conclusion

The independency of the Ricci tensor on the variables $\{x_1, x_2, x_3, x_4\}$ is a common feature of the Non-strict walker metric as seen in theorem (2.3). This results to a zero scalar curvature of the metric as shown in (2.4). The zero scalar curvature also lead to a non Einstein property of the metric. If the associated functions a, b, c are constants then the Non-strict walker metric is locally conformaly flat. There are many more properties associated with the Non-strict walker metric that need to be explored.

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Competing Interests

Authors have declared that no competing interests exist.

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