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Geometry of Projective Tensor of Viasman-Gray Manifold

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Authors' contributions

This work was carried out in collaboration between both authors. Author HMA suggested the problem and the outline of the work. Authors HMA and HGAA checked all the proofs of the theorems. Author HMA wrote the protocol and supervised the work. Author HGAA managed the analyses of the study. Author HGAA wrote the first draft of the manuscript. Author HMA managed the literature searches and edited the manuscript. Both authors read and approved the final manuscript.

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Abstract

In this paper, the Vaisman-Gray manifolds with flat projective curvature tensor are investigated. It is shown that which conditions are necessary for a Vaisman-Gray manifold with flat projective curvature tensor is a nearly Kaehler (NK) manifold, a locally conformal Kahler (LCK) manifold and an Einstein manifold. Finally, The relation between certain two special classes of almost Hermitian manifold with respect to the projective tensor has been studied.

Keywords: Almost Hermitian manifold; Viasman-Gray manifold; projective tensor.

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1 Introduction

Differential geometry has a long history as field of mathematics and yet its rigorous founda tion in the realm of contemporary mathematics is relatively new. In particular, almost Hermitian manifold occupies an important place in modern differential geometry. The notion of almost Hermitian manifold is one of the central concepts of modern mathematics and applications. Therefore, a great number of researches have been devoted to study of such structures and used more than one way to study this structure. One of these methods that does not depend on manifold itself but on a principle subfiber bundle of all complex frames which is called the adjoined G-structure space [5]. We used this method to study the projective tensor of one more specific important class of the sixteen classes of almost Hermitian manifold which is Viasman-Gray manifold (VG-manifold). This class denoted by $W_1 \oplus W_4$, where W_1 is the nearly Kähler manifold (NK-manifold) and W_4 is the locally conformal Kähler manifold (LCK-manifold).

In 1994, Kirichenko and Shchipkova [9] studied the class $W_1 \oplus W_4$under the name Viasman-Gray manifold. They found its structure equations in the adjoined G-structure space. In 1996, Kirichenko and Eshova [8] studied the conformal invariant of the class $W_1 \oplus W_4$.

The constant type of Viasman-Gray manifold has been studied by Vanhecke and Bouten [15] and Kirichenko [7]. For Viasman-Gray manifold, this notion represents a generalization of constant type of nearly Kähler manifold which is introduced by Gray [4]. In 2002, Ignatochkina [11] studied the geometric meaning of flat conformal invariant classes of Viasman-Gray manifold. In particular, it has been proved that VG-manifold of dimension greater than 4 with J-invariant conformal curvature tensor is either local conformal equivalent to the nearly Kähler manifold, or is local conformal Kähler manifold.

There were many authors studied the projective tensor. Kirichenko [6] proved that projective-recurrent K-space of dimension $n > 2$ is local symmetric or local equivalent to the product of Euclidean space and 2-dimensional Kähler manifold. Abood and Mohammed [1] proved that almost Kähler manifold is a Kähler manifold if it is a projective parakähler manifold.

2 Preliminaries

Let M be a $2n$-dimensional $(n > 1)$ smooth manifold, and the $X(M)$ be module of smooth vector fields on M, $C^\infty(M)$ be the set of smooth functions of M.

A pair $\{J, g = \langle \cdot, \cdot \rangle\}$ equipped with smooth manifold M is called almost Hermitian manifold (AH-manifold), where J is an endomorphism of tangent space $T_p(M)$ with $(J_p)^2 = -id$ and $g = <.,.>$ is a Riemannian metric on M such that it is compatible with almost complex structure of M [4].

The basis of $T_p^c(M)$ given by the form$\{\varepsilon_1, \dots, \varepsilon_2, \bar{\varepsilon}_1, \dots, \bar{\varepsilon}_n\}$ which called adapted basis, where $T_p^c(M)$ is the complexification of the tangent space $T_p(M)$ at the point $p \in M$. The corresponding its complex frame is $\{p, \varepsilon_1, \dots, \varepsilon_2, \bar{\varepsilon}_1, \dots, \bar{\varepsilon}_n\}$. The G-structure space is the principle fiber bundle of all complex frames of manifold M with structure group is the unitary group $U(n)$. This space is called an adjoined G-structure space [5].

Suppose that the indices i, j, k, l in the range $1,2, \dots, 2n$ and the indices a, b, c, d, e, f in the range $1,2, \dots, n$ and $\hat{a} = a + n$.

In the adjoined G-structure space the components matrices of complex structure J and Riemannian metric g have the following forms:

$$(J_j^i) = \begin{pmatrix} \sqrt{-1} I_n & 0 \\ 0 & -\sqrt{-1} I_n \end{pmatrix}, (g_{ij}) = \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix}, \tag{2.1}$$

where I_n is the unit matrix of order n [5].

Definition 2.1 [3]. An $AH-$manifold is called VG-manifold, if in the adjoined G-structure space, the following conditions satisfies:

$$B^{abc} = B^{-bac} \quad ; \quad B^{ab}{}_{c} = \alpha^{[a}\delta_{c}^{b]} \ ;$$

An AH-manifold is called a locally conformal Kähler manifold if $B^{\text{abc}} = 0$ and $B^{ab}{}_{c} = \alpha^{[a}\delta_{c}^{b]}$, and is called a nearly Kähler manifold if $B^{abc} = -B^{bac}$, $B^{ab}{}_{c} = 0$,

where $B^{abc} = \frac{\sqrt{-1}}{2}J^{a}_{[\hat{b},\hat{c}]}$ $\quad B^{ab}{}_{c} = -\frac{\sqrt{-1}}{2}J^{a}_{\hat{b},c}$ and $\alpha = \frac{1}{n-1}\delta F \circ J$; F is a Kähler form which defined by $F(X,Y) = < JX, Y >$, δ is a codrivative and $X, Y \in X(M)$and the bracket [ ] denote to the antisymmetric operation.

Theorem 2.2 [9]. In the adjoined G-structure space, the family of the structure equations of VG-manifold has the following forms:

1) $d\omega^{a} = \omega^{a}_{b}\Lambda\omega^{b} + B^{ab}{}_{c}\omega^{c}\Lambda\omega_{b} + B^{abc}\omega_{b}\Lambda\omega_{c}$;

2) $d\omega_{a} = -\omega^{b}_{a}\Lambda\omega_{b} + B_{ab}{}^{c}\omega_{c}\Lambda\omega^{b} + B_{abc}\omega^{b}\Lambda\omega^{c}$;

3) $$d\omega^{a}_{b} = \omega^{a}_{c}\Lambda\omega^{c}_{b} + \left(2B^{adh}B_{hbc} + A^{ad}_{bc}\right)\omega^{c}\Lambda\omega_{d} + \left(B^{ah}{}_{[c}B_{d]bh} + A^{a}_{bcd}\right)\omega^{c}\Lambda\omega^{d} + \left(B_{bh}{}^{[c}B^{d]ah} + A^{acd}_{b}\right)\omega_{c}\Lambda\omega_{d} ,$$

where$\{\omega^{i}\}$ are the components of mixture form,$\{\omega^{i}_{j}\}$ are the components of Riemannian connection of metric g, $\{A^{a}_{bcd}, A^{acd}_{b}\}$ are some functions on adjoined G-structure space and $\{A^{ad}_{bc}\}$ are system of functions which are symmetric by the lower and upper indices and are called the components of holomorphic sectional curvature tensor.

Definition 2.3 [10]. A Riemannian curvature tensor R of smooth manifold M is an 4-covariant tensor $R: T_{p}(M) \times T_{p}(M) \times T_{p}(M) \times T_{p}(M) \to \mathbb{R}$ which is defined by

$$R(X,Y,Z,W) = g(R(Z,W)Y,X),$$

where $R(X,Y)Z = \left([\nabla_{X},\nabla_{Y}] - \nabla_{[X,Y]}\right)Z$; $X,Y,Z,W \in T_{p}(M)$ and satisfies the following pro-perties:

1) $R(X,Y,Z,W) = -R(Y,X,Z,W)$;

2) $R(X,Y,Z,W) = -R(X,Y,W,Z)$;

3) $R(X,Y,Z,W) + R(X,Z,W,Y) + R(X,W,Y,Z) = 0$;

4) $R(X,Y,Z,W) = R(Z,W,X,Y)$.

Theorem 2.4 [11]. In the adjoined G-structure space, the components of Riemannian curvature tensor R of VG $-$manifold are given as follows:

1) $R_{abcd} = 2(B_{ab[cd]} + \alpha_{[a}B_{b]cd})$;

2) $R_{\hat{a}bcd} = 2\mathrm{A}^{a}_{bcd}$;

3) $R_{\hat{a}\hat{b}cd} = 2(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]})$;

4) $R_{\hat{a}bc\hat{d}} = A^{ad}_{bc} + B^{adh}B_{hbc} - B^{ah}{}_{c}B_{hb}{}^{d}$,

where $\{\alpha^a{}_b, \alpha_a{}^b, \alpha_{ab}, \alpha^{ab}\}$ are some functions on adjoined G −structure space such that

$$d\alpha_\mathrm{a} + \alpha_b\omega_a^b = \alpha_a^b\omega_b + \alpha_{ab}\omega^b,$$

And

$$d\alpha^a - \alpha^b\omega_b^a = \alpha_b^a\omega^b + \alpha^{ab}\omega_b.$$

The other components of Riemannian curvature tensor R can be obtained by the property of symmetry for R.

Definition 2.5 [13]. A tensor of type $(2,0)$which is defined as $r_{ij=} R_{ijk}^k = g^{kl}R_{kijl}$ is called a Ricci tensor.

Theorem 2.6 [11]. In the adjoined G-structure space, the components of the Ricci tensor of VG-manifold are given by the following forms:

1) $r_{ab} = \frac{1-n}{2}(\alpha_{ab} + \alpha_{ba} + \alpha_a\alpha_b);$

2) $r_{\hat{a}b} = 3\mathrm{B}^{cah}\mathrm{B}_{cbh} - \mathrm{A}_{bc}^{ca} + \frac{n-1}{2}(\alpha^a\alpha_b - \alpha^h\alpha_h) - \frac{1}{2}\alpha^h{}_h\delta_b^a + (n-2)\alpha^a{}_b).$

And the others are conjugate to the above components.

3 Main Results

Definition 3.1 [13]. A projective tensor of an AH-manifold is a tensor P of type $(4,0)$which is defined by the form:

$$P_{ijkl} = R_{ijkl} + \frac{1}{2n-1}\left(r_{ik}g_{jl} - r_{jk}g_{il}\right),$$

where, R_{ijkl} , r_{ij} and g_{ij} are respectively the components of Riemannian curvature tensor, Ricci tensor and Riemannian metric.

This tensor has properties similar to the those of Riemannian curvature tensor,

i.e. $P_{ijkl} = -P_{jikl} = -P_{ijlk} = P_{klij}$.

Lemma 3.2. In the adjoined G-structure space, the components of the projective tensor of VG-manifold are given by the following forms:

1) $P_{abcd} = 2\left(B_{ab[cd]} + \alpha_{[a}B_{b]cd}\right)$;

2) $P_{\hat{a}bcd} = 2\mathrm{A}_{bcd}^{a} - \frac{1}{2n-1}r_{bd}\delta_\mathrm{d}^\mathrm{a}$;

3) $P_{\hat{a}\hat{b}cd} = 2\left(-B^{abh}B_{hcd} + \alpha_{[c}^{[a}\delta_{d]}^{b]}\right) + \frac{2}{2n-1}r_c^{[a}\delta_d^{b]}$;

4) $P_{\hat{a}bc\hat{d}} = A_{bc}^{ad} + B^{adh}B_{hbc} - B^{ah}{}_cB_{hb}{}^d + \frac{1}{2n-1}r_c^a\delta_b^d.$

Proof.

1) For $i = a$, $j = b$, $k = c$, and $l = d$, we have
$P_{abcd} = R_{abcd} + \frac{1}{2n-1}(r_{ac}g_{bd} - r_{bc}g_{ad}).$

According to the equation (2.1), we get that

$$P_{abcd} = R_{abcd}.$$

2) For $i = \hat{a}$, $j = b$, $k = c$ and $l = d$, we have

$$P_{\hat{a}bcd} = R_{\hat{a}bcd} + \frac{1}{2n-1}(r_{\hat{a}c}g_{bd} - r_{bc}g_{\hat{a}d}),$$

$$P_{\hat{a}bcd} = R_{\hat{a}bcd} - \frac{1}{2n-1}r_{bc}\delta_d^a\ .$$

3) For $i = \hat{a}$, $j = \hat{b}$, $k = c$ and $l = d$, we have

$$P_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} + \frac{1}{2n-1}(r_{\hat{a}c}g_{\hat{b}d} - r_{\hat{b}c}g_{\hat{a}d}),$$
$$= R_{\hat{a}\hat{b}cd} + \frac{1}{2n-1}\left(r_c^a\delta_d^b - r_c^b\delta_d^a\right)$$
$$= R_{\hat{a}\hat{b}cd} + \frac{2}{2n-1}r_c^{[a}\delta_d^{b]}\ .$$

4) For $i = \hat{a}$, $j = b$, $k = c$, and $l = \hat{d}$, we have

$$P_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} + \frac{1}{2n-1}(r_{\hat{a}c}g_{b\hat{d}} - r_{bc}g_{\hat{a}\hat{d}})$$
$$= R_{\hat{a}bc\hat{d}} + \frac{1}{2n-1}r_c^a\delta_b^d\ .$$

Definition 3.3. An AH-manifold is called a projective flat if the projective tensor is equal to zero.

Theorem 3.4. Suppose that M is VG- manifold with flat projective tensor, if M is a manifold of flat Ricci tensor then M is NK- manifold.

Proof. Suppose that M is VG-manifold with flat projective tensor.

Making use of Lemma 3.2. and Definition 3.3, it follows that

$$R_{\hat{a}bc\hat{d}} + \frac{1}{2n-1}r_c^a\delta_b^d = 0.$$

According to the Theorem 2.4, we obtain

$$A_{bc}^{ad} + B^{adh}B_{hbc} - B^{ah}{}_cB_{hb}{}^d + \frac{1}{2n-1}r_c^a\delta_b^d = 0.$$

Since M is manifold of flat Ricci tensor, then we deduce

$$A_{bc}^{ad} + B^{adh}B_{hbc} - B^{ah}{}_cB_{hb}{}^d = 0.$$

Symmetrizing and antisymmetrizing by the indexes(a, d), it follows that

$$-B^{ah}{}_cB_{hb}{}^d = 0\ .$$

Contracting the last equation by the indexes (a, b) and (d, c), we have

$$-B^{ah}{}_dB_{ha}{}^d = 0\ .$$
$$B^{ah}{}_dB_{ha}{}^d = 0 \Leftrightarrow \sum\left|B^{ah}{}_d\right|^2 = 0 \qquad \Leftrightarrow\ B^{ah}{}_d = 0\ .$$

According to the Definition 2.1 we get that M is NK- manifold.

Definition 3.5 [12]. A Riemannian manifold is called an Einstein manifold, if the Ricci tensor satisfies the equation $r_{ij} = cg_{ij}$, where c is an Einstein constant.

Definition 3.6 [2]. An AH-manifold has J-invariant Ricci tensor, if $J \circ r = r \circ J$.

The following lemma gives the necessary and sufficient condition for which an AH-manifold has J-invariant Ricci tensor.

Lemma 3.7 [14]. An AH-manifold has J-invariant Ricci tensor if and only if, in the adjoined G-structure space we have $r_b^{\hat{a}} = r_{ab} = 0$.

Theorem 3.8. Suppose that M is VG-manifold with flat projective tensor and J-invariant Ricci tensor. Then $A_{ac}^{ad} = c\delta_c^d$ if and only if, M is an Einstein manifold.

Proof. Suppose that M is VG-manifold with a flat projective tensor.

By using Lemma 3.2., definition 3.3. and theorem 2.4., we have

$$A_{bc}^{ad} + B^{adh}B_{hbc} - B^{ah}{}_c B_{hb}{}^d + \frac{1}{2n-1} r_c^a \delta_b^d = 0 \ . \tag{3.1}$$

By symmetrization the equation (3.1) by the indices (h, b) we get

$$A_{bc}^{ad} + \frac{1}{2}(B^{adh}B_{hbc} + B^{adh}B_{bhc} - B^{ah}{}_c B_{hb}{}^d - B^{ah}{}_c B_{bh}{}^d) + \frac{1}{2n-1} r_c^a \delta_b^d = 0.$$

Antisymmetrizing the last equation by the indices (h, b) we obtain

$$A_{bc}^{ad} + \frac{1}{2n-1} r_c^a \delta_b^d = 0 \ . \tag{3.2}$$

Suppose that M is an Einstein manifold, so the equation (3.2) becomes

$$A_{bc}^{ad} + \frac{e}{2n-1}\left(\delta_c^a \delta_b^d\right) = 0 \ . \tag{3.3}$$

Contracting the equation (3.3) by indexes (a, b), it follows that

$$A_{ac}^{ad} = \frac{-e}{2n-1}(\delta_c^a \delta_a^d) \ ,$$

$$A_{ac}^{ad} = \frac{-e\delta_c^d}{2n-1},$$

$$A_{ac}^{ad} = c\delta_c^d \ ,$$

where $c = \frac{-e}{2n-1}$ is the Einstein constant.

Conversely, by contracting (3.2) by the indexes (a, b) we get

$$A_{ac}^{ad} + \frac{1}{2n-1}(r_c^a \delta_a^d) = 0,$$

$$A_{ac}^{ad} + \frac{r_c^d}{2n-1} = 0,$$

$$\frac{r_c^d}{2n-1} = \frac{e}{2n-1}\delta_c^d,$$

$$r_c^d = e\delta_c^d.$$

Since M is J-invariant Ricci tensor, then M is an Einstein manifold.

Theorem 3.9. Suppose that M is VG-manifold with flat projective tensor and J-invariant Ricci tensor, if M is an Einstein manifold then M is LCK-manifold.

Proof. Suppose that M is VG-manifold with flat projective tensor.

Making use of Lemma 3.2., definition 3.3 and theorem 2.4, it follows that

$$A_{bc}^{ad} + B^{adh}B_{hbc} - B^{ah}{}_{c}B_{hb}{}^{d} + \frac{1}{2n-1}r_c^a\delta_b^d = 0\ .$$

Contracting the above equation by the indexes (a, b), we obtain

$$A_{ac}^{ad} + B^{adh}B_{hac} - B^{ah}{}_{c}B_{ha}{}^{d} + \frac{1}{2n-1}r_c^a\delta_a^d = 0\ .$$

Since M is an Einstein manifold, then according to the definition 3.5, we have

$$A_{ac}^{ad} + B^{adh}B_{hac} - B^{ah}{}_{c}B_{ha}{}^{d} + \frac{e}{2n-1}\delta_c^a\delta_a^d = 0\ .$$

By using the Theorem 3.8, we obtained

$$\frac{-e\delta_c^d}{2n-1} + B^{adh}B_{hac} - B^{ah}{}_{c}B_{ha}{}^{d} + \frac{e}{2n-1}\delta_c^d = 0\ ,$$

$$B^{adh}B_{hac} - B^{ah}{}_{c}B_{ha}{}^{d} = 0\ .$$

Since M is VG-manifold, then we have

$$-B^{adh}B_{hca} - B^{ah}{}_{c}B_{ha}{}^{d} = 0\ .$$

Symmetrizing and antisymmetrizing the last equation by the indexes(a, h), it follows that

$$-B^{adh}B_{hca} = 0\ .$$

Contracting the above equation by the indexes (d, c), we deduce

$$B^{adh}B_{adh} = 0 \Longrightarrow \bar{B}_{adh}B_{adh} = 0 \Longrightarrow \sum_{a,d,h}|B_{adh}|^2 = 0 \Leftrightarrow B_{adh} = 0\ .$$

Therefore, by the definition 2.1 we get that M is LCK-manifold.

There are three special classes of almost Hermitian manifold which are embodied in the following lemma.

Lemma 3.10 [14]. In the adjoined G −structure space, an AH- manifold is manifold of class:

R_1 if and only if, $R_{abcd} = R_{\hat{a}bcd} = R_{\hat{a}\hat{b}cd} = 0$,

R_2 if and only if, $R_{abcd} = R_{\hat{a}bcd} = 0$,

R_3 (*RK*-manifold) if and only if, $R_{\hat{a}bcd} = 0$.

Similarly, we can give their corresponding according to the projective tensor by the following lemma.

Lemma 3.11. In the adjoined G −structure space, an AH- manifold is manifold of class:

PR_1 if and only if, $P_{abcd} = P_{\hat{a}bcd} = P_{\hat{a}\hat{b}cd} = 0$,

PR_2 if and only if, $P_{abcd} = P_{\hat{a}bcd} = 0$,

PR_3(PRK −manifold) if and only if, $P_{\hat{a}bcd} = 0$.

Theorem 3.12. Suppose that M is VG-manifold, the classes R_3 and PR_3 are coincide if and only if M is J-invariant Ricci tensor.

Proof. Suppose that R_3 and PR_3 are coincide, then we have

$$-\frac{1}{2n-1} r_{bc}\delta_d^a = 0,$$
$$r_{bc} = 0.$$

Therefore, M is J-invariant Ricci tensor.

Conversely, Suppose that M is J-invariant Ricci tensor.

Making use of Lemma 3.2., it follows that

$$P_{\hat{a}bcd} = R_{\hat{a}bcd} - \frac{1}{2n-1} r_{bc}\delta_d^a .$$

Since M is J-invariant Ricci tensor, then according to the Lemma 3.7 we get

$$P_{\hat{a}bcd} = R_{\hat{a}bcd} .$$

Therefore, R_3 and PR_3 are coincide.

4 Conclusion

This paper studied the geometrical properties of projective curvature tensor of Viasman-Gray manifold. We found out the necessary conditions of flatness of projective tensor, in particular, we found an interesting theoretical physical application.

Competing Interests

Authors have declared that no competing interests exist.

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